American Journal of Multidisciplinary Research and Innovation (AJMRI)

Nonpre-Emptive Integer Nonlinear Goal Programming Model for Multi-Item Inventory Problem: Case Study of a Car Retail Centre in Lagos State.

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ABSTRACT

Most of the real-world optimization problems involve multiple objectives with constraint resources. Retailers oftentimes anticipate demand, they stock, and maintain warehouses with different variety of products at high cost to meet prospecting customers' demand. In this paper, a Non-preemptive Integer Nonlinear Goal Programming (NINGP) model was developed for obtaining Economic Order Quantities (EOQ) of multi-item inventory problems that satisfy the multiple and conflicting objectives of the Decision Maker (DM). The particular case considered was that of a motor vehicle dealer who sells 10 brands of Tokunbo vehicles and wants to determine the EOQ for each brand such that the deviations from the aspiration level are minimized. Using LINGO 17.0 Software to solve the NINGP model, the EOQ allocated to each brand type from 1 through 10 are 2, 2, 5, 2, 2, 3 3, 3, 3, and 3 cars respectively. The optimal number of cars was 28 with the associated cost of ₦53,825,915. Compared to the estimated budget of ₦60,000,000, the NINGP approach was able to achieve a 10% (₦6,174,085) below budget. With proper modifications considering the associated constraints, related inventory problems can be solved using the NINGP model.

INTRODUCTION

According to sayed et al. (2014) non-preemptive integer nonlinear goal programming (NINGP) helps to solve problems associated with multi-item inventory decision-making problems. For the most part, real-world optimization problems involve multiple objectives with constraint resources. Retailers oftentimes anticipate demand, therefore stock, and maintain their warehouses with different varieties of products at a high cost to meet prospecting customers' demand. Multiple items or products are stored in most shops or warehouses to increase profitability, and competition and attract sales from customers with different choices. However, keeping these inventories is quite challenging as their values keep increasing owing to increased cost of importation, fluctuating exchange rates, and changing economic policies (Sayed et al., 2014). Adeyemi (2010) opined that "to maintain a balance of the conflicting economics, an effective inventory management is required to determine the number inventories in stock at minimum cost. Proper management policies, and control measures must also be in place to monitor and replenish these inventories (Nsikan et al., 2015). Keeping huge and numerous inventories can lead to an expanded running cost, low benefits and tall working capital prerequisites whereas investment in little inventory can lead to break-ups in the organizational handle, loss of customers, and a greatly reduced profit margin (Prabha, and Suchita, 2016). Amini et al. (2017) state that "the major concern in real-life decision-making situations involve multiple criteria (attributes or objectives) rather than single criteria,". These objectives are conflicting and are best approached using a goal programming analytical framework. Goal programming is an operation research technique useful for achieving simultaneously multiple goals with constrained resources (Ajayi-Daniels, 2019). The aim of goal programming (GP) is to find an optimum solution out of a set of feasible solutions that satisfy the real-life constraints and comes in a closed form to the decision-makers stated target value (goals).

According to Amini, et al. (2017), Goal Programming is used to analyze all of the objectives with various achievement relations to discover an acceptable solution by reducing deviations from expected values. It also analyzes how much a proposed optimal solution deviates from each target though there are deviation variables defined, for each pair of stated goals (Moumita & De, 2016). This concept has enjoyed significant applications by many researchers over time, in solving different inventory problems with multiple constraints resources. According to Aouni & Kettani (2001), the increase in the use of goal programming was simply because it is exceptionally simple to comprehend and applied in many areas, such as the manufacturing, production, retail shops, transportation, medicine, agriculture, academic institutions, and construction companies alike. In the study of Ajayi-Daniels (2019), a goal-programming model was developed to optimize resources in a fashion firm where goals, were prioritized according to importance. The result-achieved bases on the priority level show a reduction in the overtime hours from 10hours to 8 hours, which is the optimum time, and a target profit margin was achieved. Also with efficient use of resources the set goal of 3 garments per day was achieved. Kliestik et al. (2015), developed a unique GP model that management companies with numerous plans can use to implement a strategic goal. Yahia-Berrouiguet & Tissourassi (2015), proposed a goal-programming model for the allocation of

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time and cost to three different projects with preemptive goals. They discussed the illogical allocation of zero (0) time to project planning and concluded that a project is bound to fail if the planning phase is not given proper consideration with an allocation of time. Jana & Das (2016) previewed a multi-item two-warehouse inventory model with a nested discount on unit cost and inventory costs over a fixed period. The proposed a multi-objective genetic algorithm with a varying populations (MOGAVP) for multi-item two-warehouse inventory problems with nested price breaks. The optimal shipments, lot size of the two warehouses, shipment size, and maximum profit are determined by maximizing the profit function. Huseyinov & Bayrakdar (2019) developed Non-Dominated Sorting Genetic Algorithm-III (NSGA-III) and Strength Pareto Evolutionary Algorithm (SPEA2) for solving a simulated multi-objective single-period multi-item inventory problem. The study proved that the SPEA-II outperformed NSGA-III with spacing, generational distance, and hypervolume as performance evaluation metrics.

Waliv et al. (2020) proposed a multi-objective, multi-item fuzzy stochastic inventory model for deteriorating items under limited storage space as well as capital investment. Their proposed model considered some parameters to be vague and random, by representing the vagueness parameters membership function and randomness parameters as a probability distribution. In their study, fuzzy nonlinear programming (FNLP) and intuitionistic fuzzy optimization (IFO) methods to solve the multi-objective fuzzy stochastic inventory model were developed. Their study showed that FNLP performed better than IFO in the area of minimizing shortage costs. Das (2020) developed the multi-item inventory model in the fuzzy environment. The formulated Multi-objective inventory problem was used to solve by different approaches such as the Geometric Programming (GP) approach, the Fuzzy Programming Technique with Hyperbolic Membership Function (FPTHMF), Fuzzy Nonlinear Programming (FNLP) technique, and Fuzzy Additive Goal Programming (FAGP) technique.

The study of Teymouri et al. (2020) presented a multi-item supplier selection, order allocation, and pricing method with stochastic demand and purchase from the suppliers who provide goods with all-unit discount. They developed a mixed nonlinear integer single-objective model and applied a genetic algorithm to solve the model. The result showed that increasing the variance of the random variable of demand results in decreasing the profit level. The study also shows that the genetic algorithm is considered an appropriate alternative to solve a mixed nonlinear integer model of supplier selection, order allocation, and pricing. Cárdenas-Barrón et al. (2021) considered the multi-item inventory lot-sizing problem with supplier selection and proposed a facility location extended formulation with the preprocessed based on the cost structure and describe new valid inequalities in the original space of variables. An effective MIP (mixed

integer programming) heuristic using the extended formulation was the model to implement the proposed formulation. In the study, the results showed that the preprocessing approach can significantly reduce the size of the formulation and also allow both an increase in the number of instances solved to optimality within the time limit and a reduction in the average time to solve them. The performance of the approaches can vary considerably under different cost structures.

Meanwhile, little/ less attention is on the inventory of foreign (Tokubo) car retail business. Therefore, this paper developed the NINGP model to determine the EOQ required in a Car Retail Centre in other to achieve the Decision Maker's competitive priorities with stated constraints.

MATERIALS AND METHOD
The approach of the Non-preemptive goal programming involves establishing a specific numerical value (target) for the objectives of the Decision Maker (DM). Deviations from these targets are not desirable, therefore the DM seeks a compromise solution.

Brief Description of the Problem
A Second-hand (Tokunbo) vehicle dealer in the metropolitan city of Lagos-state wishes to determine the economic order quantity (EOQ), for the vehicles, he sells. He sells ten (10) different and affordable brand of vehicles with a yearly budget of sixty million, three hundred and fifteen thousand naira's (N60, 315,000.00). The available retail space has the capacity for 25 or more Second-hand cars. Due to the retail space and other possible constraints, the dealer wishes to determine the optimal mix of these vehicles that will fill up the retail space and still be able to meet prospecting customers' demand.

Assumptions of Model
The model assumptions are:
   i. The inventory system involves multiple items
   ii. For every replenishment, order for a single delivery is made
   iii. There is a constant Lead time
   iv. Shortages are not allowed
   v. Discounts are not allowed for any item ordered
   vi. Equal space is available for all items ordered
   vii. Inventory costs (Carrying cost and ordering cost) are known
   viii. Demand is known and constant.
   ix. The inventory parameters are preset and constant
   x. The goals are of equal weight as well as the deviations from the target
   xi. The goals are of equal importance

Mathematical Formulations
The general model is of the form

$$\text{Minimize:} \qquad T_{ic} = \sum_{i=1}^{n} \left[S_{oc_i} O_{qi} + \frac{S_{ci}}{q_i} \right] \dots \dots \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} a_{ij} O_{qi} \leq b_j \text{ where } 1 \leq j \leq m$$

$$\sum_{i=1}^{n} O_{qi} \leq S_c, \quad where \quad 1 \leq i \leq n, \ q_i integer \ \forall i$$

Table 1.1: Notations with Description

Indices	Description
i	Integer index for items (i= 1, 2, 3 …n)
j	constraints Index (j = 1, 2 …m)
a_{ij}	The average investment per unit of item i
oq_i	The decision variable (ordered quantity of item i)
b_j	The right-hand side value associated with constraint j
d_i^-, d_j^-	The negative deviation variables of the NINLGP from the item (i) and constraint (j) (underachievement)
d_i^+, d_j^+	The positive deviational variable of the INLGP from the ith goal and constraint j (over-achievement)
S_{oci}	Sum of ordering cost and holding cost for ith item
T_{ic}	Total inventory cost
S_{ci}	Setup cost for item i
G_i	The target value for the item i
S_c	Storage capacity

Goal Formulation

The NINGP model objective and constraints are taken as the model goals with the introduction of deviational variables.

Total Inventory Cost Goal

Deviational variables added to the inventory cost to give:

$$\sum_{i=1}^{n}\left[S_{oci}O_{qi} + \frac{S_{ci}}{O_{qi}}\right] + d_i^- - d_i^+ = G_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Investment Goal

With the deviational variables the targeted investment goal gives:

$$\sum_{i=1}^{n} a_{ij}O_{qi} + d_j^- - d_j^+ = b_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Inventory Space Goal

The targeted inventory space becomes;

$$\sum_{i=1}^{n} O_{qi} + d_i^- - d_i^+ = S_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Weighted Average Structure

Accordingly, the weighted average structure of the Non-preemptive Integer Nonlinear Goal Programming (NINGP) is stated as;

Find $O_q^-(O_{q1}, O_{q2}, \ldots O_{qi})$ to Minimize:

$$\left[w_i(d_i^- + d_i^+) + w_j\left(d_j^- + d_j^+\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Subjected to

$$\sum_{i=1}^{n}\left[S_{oci}q_i + \frac{S_{ci}}{O_{qi}}\right] + d_i^- - d_i^+ = G_i \qquad 1 \leq i \leq n \ldots\ldots\ldots\ldots(6)$$

$$\sum_{i=1}^{n} a_{ij}O_{qi} + d_j^- - d_j^+ = b_j \qquad 1 \leq j \leq m \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\sum_{i=1}^{n} O_{qi} + d_i^- - d_i^+ = S_c \qquad O_{qi} \ (integer \ \forall \ i) \ldots\ldots\ldots\ldots\ldots(8)$$

Where, $d_i^-, d_j^-, d_i^+, d_j^+ (integer \ \forall \ i, j) w_i = w_j = 1$, since all goals are presumed to be of equal importance to the DM.

Model Application

The cost detail for each brand of "Tokunbo" car is tabulated below.

Table 1.2: Cost details for each brand of cars

Car Type		(Ci)(₦) '000	Setup cost (S)(₦) '000	Target value (G)(N) '000	Average Investment (a_ij)(₦) '000
O_{q1}	Toyota Camry (2004)	3963	2600	10000	7613
O_{q2}	Toyota Corolla (2006)	4325	2463	10000	8013
O_{q3}	Toyota Corolla (2004)	1550	2325	8000	4225
O_{q4}	Peugeot 307 (2002)	2688	2380	7000	5768
O_{q5}	Peugeot 307 (2004)	3525	2500	9000	7163
O_{q6}	Honda Accord (2004)	2550	2500	8200	5750
O_{q7}	Nissan Quest	2162	2225	8000	5000
O_{q8}	Honda Accord (2003)	2505	2363	9000	5629
O_{q9}	Honda Civic (2003)	2437	2363	9000	5509
O_{q10}	Toyota sienna (2003)	2637	2325	9000	5645
	Total	28,342	22,044	87,200	60,315

Inventory Cost goals

The inventory cost goals for brands 1 through 10 respectively are presented in equations (11–20) below using the data in Table 2:

$$3963O_{q_1} + \frac{2600}{o_{q_1}} + d_1^- - d_1^+ = 10000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

$$4325O_{q_2} + \frac{2463}{o_{q_2}} + d_2^- - d_2^+ = 10000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$$1550O_{q_3} + \frac{2325}{o_{q_3}} + d_3^- - d_3^+ = 8000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$2688O_{q_4} + \frac{2380}{o_{q_4}} + d_4^- - d_4^+ = 7000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

$$3525O_{q_5} + \frac{2500}{o_{q_5}} + d_5^- - d_5^+ = 9000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

$$2550O_{q_6} + \frac{2500}{o_{q_6}} + d_6^- - d_6^+ = 8200; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

$$2160O_{q_7} + \frac{2225}{o_{q_7}} + d_7^- - d_7^+ = 8000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

$$2505O_{q_8} + \frac{2363}{o_{q_8}} + d_8^- - d_8^+ = 9000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

$$2437O_{q_9} + \frac{2363}{o_{q_9}} + d_9^- - d_9^+ = 9000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$2637O_{q_{10}} + \frac{2325}{o_{q_{10}}} + d_{10}^- - d_{10}^+ = 9000; \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Investment constraint

Since the cost of each vehicle ordered is fixed the equation becomes:

$$(76130_{q_1} + 80130_{q_2} + 52250_{q_3} + 57680_{q_4} + 71630_{q_5} + 57500_{q_6} + 50000_{q_7} + 56290_{q_8} + 55090_{q_9} + 56450_{q_{10}}) + d_{11}^- - d_{11}^+ = 165062 \quad \text{.................(21)}$$

Inventory constraint

The total carrying capacity of the warehouse is 30 cars. Thus, the goal equation appears a

$$(0_{q_1} + 0_{q_2} + 0_{q_3} + 0_{q_4} + 0_{q_5} + 0_{q_6} + 0_{q_7} + 0_{q_8} + 0_{q_9} + 0_{q_{10}}) + d_{12}^- - d_{12}^+ = 30 \quad \text{.................(22)}$$

RESULTS AND DISCUSSION

The NINGP Model was tested and the result is provided in table 2.

Table 2: The optimal mix of various types of cars and their respective deviations

Car type	Quantity (EOQ)	Positive (+)/Negative (-) Deviation (N) from the budget
O_{q1}	2	-1204275
O_{q2}	2	-1348312
O_{q3}	5	529619
O_{q4}	2	-760805
O_{q5}	2	-1054150
O_{q6}	3	830042
O_{q7}	3	-696021
O_{q8}	3	-815128
O_{q9}	3	-790410
O_{q10}	3	-864645
Total	28	-6174085

Table 2 shows that the dealer can stock up the car Centre with 10 different types of cars occupying 93% of the inventory space. For instance, at the retail Centre two types of Toyota Camry (2004), Toyota Corolla (2006), Peugeot 307 (2002) and Peugeot 307 (2004) must always be on display. There must always be five types of Toyota Corolla (2004) on display and 3 types each for the following car types; Honda Accord (2004); Nissan Quest; Honda Accord (2003); Honda Civic (2003); Toyota sienna (2003). Also the table shows an under-achieved budgeted goals for each car type. The solution saved the decision-maker the sum of ₦6,174,085, which represent 10.3% of the decision maker's annual budget estimate, which can also purchase two or three Tokunbo cars. The optimal cost of the cars is ₦53,825,915 as against the initial cost of ₦60,315,000 in table 2 from the saved cost of ₦6,174,085). The highlighted table 2 is supposed to be written is table 1

CONCLUSION

This paper presents the application of the Nonpre-emptive Integer Nonlinear Goal Programming (NINGP) model and the procedure for solving a multi-item inventory problem in other to obtain the EOQ required to achieve the aspirations of a local car dealer. All conflicting objectives and constraints were taken into consideration and with the use of an optimization tool in LINGO 17.0 software, an optimal solution was obtained. Most Tokunbo car dealers used to display several types of cars in any available area, without properly analyzing the best mix of different car types to display and their associated costs. This makes it harder for car dealerships to achieve their objectives. They will benefit from the proposed NINGP model since it is sufficiently adaptable to meet related situation-specific objectives and constraints when the goal is to minimize costs and deviations from defined goals. Hence the NINGP model can be used analytically to solve related multi-item inventory management problems. There is to test the model for a large number of foreign "Tokunbo" and types of cars. Further study may suggest using other mathematical programming models like, stochastic and dynamic programming models, etc., to solve multi-item inventory foreign (Tokunbo) cars problem to obtain the EOQ. Also, a combined optimization algorithm and mathematical programming may be considered or involved to solve the same problem.

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