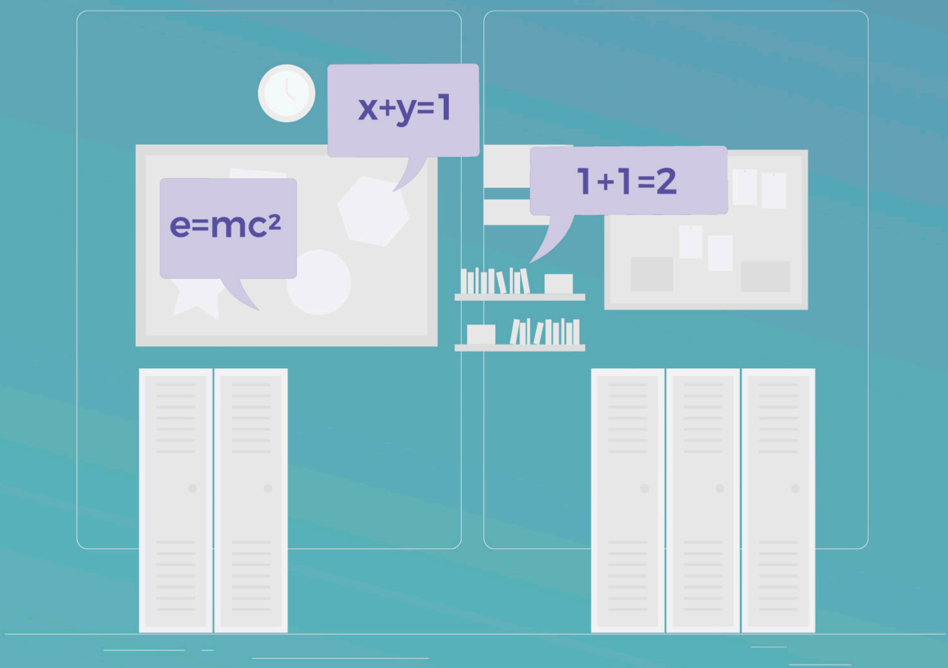
# American Journal of MATHEMATICS AND PHYSICS (AJMP)

VOLUME 1 ISSUE 1 (2026)




PUBLISHED BY
E-PALLI PUBLISHERS, DELAWARE, USA

American Journal of
Mathematics and Physics (AJMP)

Volume 1 Issue 1, Year 2026
https://journals.e-palli.com/home/index.php/ajmp

# Effect of changing the polynomial parameters of the vortex Laguerre beam on the behavior of SER

Ali A. Dheyab[1*], Asmaa M. Aubaid [2], Ekhlas Kadhum Hamza[3], Hamid Sh. Aldulaimi[4]

**Article Information**

**Received:** October 09, 2025

**Accepted:** January 08, 2026

**Published:** May 25, 2026

**Keywords**

*Laguerre Gaussian Vortex beam, Polynomial Parameters, SER, Turbulence*

## ABSTRACT

The symbol error rate (SER) is a significant parameter in the optical communication field as it demonstrates how well a system is capable of encoding data over an optical channel. Reducing the effects of SER is one of the primary objectives of designing modern optical systems since numerous variables can lower the performance, like noise and various forms of optical distortion. Laguerre polynomials can be used as a mathematical method of enhancing system reliability. This study analyzes the influence of various changes in the parameters of the polynomials that constitute the Laguerre-Gaussian vortex beam (LGVB) on the SER as the beam propagates in weak, moderate, and strong turbulence. The study explores the nature of parameter combinations to affect the intensity pattern of the beam and the resulting SER using numerical simulations. The results indicate that some combinations of parameters work significantly better than others and this results in a significant decrease in symbol errors. SER at (3,0) and (2,0) have the lowest value in the high turbulence because their intensity profiles are spatially dispersed. An effective non-hardware-altered mitigation strategy of LGVB to increase polynomial parameters improves strong SER in the turbulent FSO systems. The results of the simulation of (n=3, m=0) indicate that the suggested beam setup gives the minimum SER compared to other Laguerre Gaussian Vortex beams with various values of radial index (n), and topological charge (m) under the same turbulence conditions ($Cn^2 = 10^{-12}$ $m^{-2/3}$). This study offers practical information for the design of turbulence-resistant optical links, especially in satellite-to-ground and long-distance land communication.

## INTRODUCTION

The literature can be divided into three categories whereby the former category is about adaptive optical systems (AO) which uses the idea of wavefront sensing and correction to rectify optical disturbances (Wang *et al.*, 2018). Mackey and Dainty (2005) demonstrated that adaptive optics (AO) is effective in circumstances where there is much turbulence when the light intensity changes, but Schwartz *et al.* (2009) demonstrated that it is difficult to correct changes in phase when disturbances are of varying intensity. However, AO systems have practical implementations like constriction of aperture size (Stotts *et al.*, 2010) and cost and complexity trade offs (Sharma, 2012). In terms of modulation and detection techniques, coherent detection and advanced types of modulation such as ASK, PSK, and FSK can be used to minimize the bit error rate (BER) even in adversarial circumstances. For instance, QPSK modulation works well in underwater-air connections when combined with AO. For example, QPSK modulation achieves an effective apparent error rate (SER) in underwater-air links when paired with AO (Li & Cvijetic, 2015).

It has been demonstrated in previous studies, including Yang *et al.* (2022), that engineered and higher-order Laguerre–Gaussian (LG) modes possess the ability to cope with atmospheric turbulence. They presented numerical and experimental data which revealed that the appropriate choice of radial and azimuthal indices not only leads to an extremely low mode crosstalk but also offers a considerable decrease in symbol- and bit-error probabilities compared to conventional Gaussian beams. As an instance, Optics Express and the studies associated with it showed that when in the presence of weak to moderate turbulence, the application of mode engineering or diversity combining resulted in decreased BER and enhanced mode purity for LG beams; thus, suggesting that the (n,m) mode structure is a usable design parameter for FSO links. Recent advancements, like 100 Gbit/s multimode orbital angular momentum (OAM) transmission, show the promise of combining different methods.

The other category includes a phase screen and computational methods. Wave front sensor configuration and random phase screen (RPS) modeling have emerged as cost-effective alternatives to hardware-intensive AO (Gregory *et al.*, 2013). The simulation process was used to address amplitude and phase distortions by constructing a turbulence model using the same approach as in Abdul (2025). However, their impact on reducing the signal error rate (SER) in practical FSO systems remains under investigation. Most studies have looked at improving BER or using AO for phase correction, but

[1] Department of Electronics and Communication Engineering, Al-Nahrain University, Iraq
[2] Ministry of Higher Education and Scientific Research, Scientific Research Commission, Office of President of Scientific Research Commission, Baghdad, Al-Jadriya, Iraq
[3] Control and Systems Engineering Department, University of Technology. Baghdad. Iraq.
[4] Ministry of Higher Education and Scientific Research, Scientific Research Commission, Information and Communication Technology Research Center, Baghdad, Al-Jadriya, Iraq
[*] Corresponding author's e-mail: alia.dheyab@researchpublication.co

there are not many that focus on optimizing SER for signals affected by disturbances and low-cost solutions (like RPS) that balance performance and expense. Recent advancements in RPS algorithms coupled with quicker phase-screen generation have turned large-scale Monte Carlo studies into a practical undertaking, thus allowing for trustworthy comparisons among different beam families and turbulence levels; for example, Jiang *et al*. (2023) demonstrated that the RPS workflows optimized not only preserved Kolmogorov statistics but also brought down the computational cost, which is a conditional factor to the application of high-fidelity SER estimation over numerous (n,m) pairs.

Several studies have been published over the past few years that have demonstrated the impact of atmospheric disturbances on the propagation of light beams (Nossir *et al*., 2021; Hricha *et al*., 2022). The main objective of this work is to reduce the symbolic error rate in optical communication channels affected by the atmosphere by adjusting the mathematical parameters of the beam, instead of using expensive optical systems. The study can provide a new point of view, which excludes the use of sophisticated corrective mechanisms since an enhancement of SER is achieved by modifying the beam parameters of the LG and places a new and straightforward direction to creating the more effective and cost-efficient technologies in optical communications. This is made unique by the fact that it introduces a new method which is based on the choice of the (n, m) parameters as an efficient means of enhancing the performance of the LG beams in the presence of disturbance which is a trend that the earlier research lacked its weight in the sphere of free optical communications.

Complementary experimental demonstrations of high-capacity OAM/engineered-beam links (including lab demonstrations exceeding 100 Gbit/s over short distances) underline the practical relevance of modal design; these experiments also highlight that combining mode engineering with signal-processing (e.g., MIMO demultiplexing or neural-network compensation) can further reduce symbol-error rates in turbulent channels, suggesting a hybrid strategy where beam-parameter optimization can be paired with affordable digital compensation for best results (Doohwan *et al*., 2022). Thus, this study evaluates the SER performance of an optimized FSO system using the random phase screen (RPS) method. In comparison to previous studies, this research quantifies the maximum reduction in signal error rate (SER) due to the RPS method under the different turbulence conditions, compares its performance with that of the traditional adaptive optics (AO) and modulation techniques, and finally, gives advice on RPS application in an economically viable FSO system configuration. The structure of an article is arranged as follows, Section II The theoretical aspect of the work is presented, including a description of the expression of vortex Laguerre beam and the field intensity in the transmitter and receiver. Section III presents the calculation results with a discussion of them, and Section IV concludes with a conclusion.

**Research Questions**

RQ1: How do changes in the Laguerre–Gaussian beam parameters (n and m) affect the beam's intensity distribution?

RQ2: How do different (n, m) combinations influence the symbol error rate (SER) under varying turbulence levels?

## MATERIALS AND METHODS

In this section, it will explain some of the theoretical terms and scientific models related to optical communication techniques, such as types of beams, transmitters, channel models, receivers, etc.

**Expression of vortex Laguerre beam.**

The Laguerre polynomial is a mathematical function that represents solutions to specific differential equations. It finds application in a number of areas of mathematics, such as signal processing, mathematical fields, and linear systems analysis (Alam *et al*., 2024). It can be effectively used in optical communications to enhance the performance of the system by ensuring that SNR is enhanced and SER is minimized, which is in focus of this study. This, further, results in high-quality signals, which further enhance the work of the transmission system. A Laguerre-type vortex beam is described in this section, and its amplitude in the strong turbulence environment and corrected is considered, as well as its SER performance. This beam is defined with the help of a radial coordinate, i.e., (s, ∅). The Laguerre-Gaussian mode is the commonest of the various forms of optical vortex beams that have been mentioned in literature (Cheng *et al*., 2009; Eyyuboğlu *et al*., 2011). Its mathematical form is typically expressed as:

$$u\,(s,\emptyset) = \left(\frac{s}{\propto_s}\right)^m \exp\left(-\frac{s^2}{\propto_s^2}\right) \mathrm{L}_n^m\left(\frac{s^2}{\propto_s^2}\right) \exp(jm\emptyset) \qquad (1)$$

Where: $(s_x,s_y)$ are the transverse source plane coordinates $s=s_x^2+s_y^2$ , and u (s,∅)will become up plainly truncated because of the limited dimensions of the source plane. Therefore, $L_n^m$; It corresponds to a Laguerre polynomial, where (n) denotes the order or degree of the polynomial function (radial index that influences the number of intensity rings), $\propto_s$ is the radius of the waist of a Gaussian beam, and (m) is the topological charge of the optical vortex. The radial index influences the behavior of a beam under specific conditions, which include its capability of resisting turbulence and its capability of recovering itself following being obstructed or scattered. The radial strength of a Laguerre-Gauss (LG) beam is given by its radial index (n) which dictates the size of the concentric rings of the beam. The radial index, then, influences the behavior of a beam in some circumstances, including its resistance to turbulence and its tendency to reform itself on being blocked or scattered and the value of (n) should be raised when in strong turbulence.

On the other hand, to reduce the symbol error rate (SER), it will do so by increasing the radial index (n) of the Laguerre-Gaussian Vortex Beam (LGVB), thereby improving the performance of the optical transmission system, especially in the presence of atmospheric turbulence. Therefore, higher refractive indices lead to greater self-reconstruction capability and signal strength, which in turn improves the performance of the optical communication system (Zeng *et al*., 2019). The "azimuthal of Laguerre" can be more precisely described as the azimuthal component of mathematical solutions involving associated Laguerre polynomials, where this angle determines the orbital angular momentum (OAM) of a system. However, the azimuthal component causes the helical phase change of the light beam along the wave front; for example, when m=4, the phase changes around the axis every $8\pi$. But when m=0, there is no angular phase; therefore, the beam is a normal Gaussian beam with no phase rotation. Figure 1 shows Laguerre-Gaussian (LG) beams with two modes, explaining the azimuthal and radial indices (Mourka, 2025).

$$L_n^m(r,\emptyset,z)=\frac{\propto_s}{\omega(z)}\exp\left(\frac{izr^2}{z_0\omega(z)^2}+i(2n+m+1)\gamma(z)\right)L_n^m\left(\frac{\omega r}{\omega(z)},\emptyset,\right) \quad (2)$$

Where: $\omega(z)=\omega[1+(z/z_0)^2]^{(1/2)}$, the $\gamma(z)=\arctan(z/z_0)$, $\gamma(z)$ is the Gough phase, $z_0$ is represented by a Rayleigh range, and k wavenumber of light (Plick & Krenn, 2015).

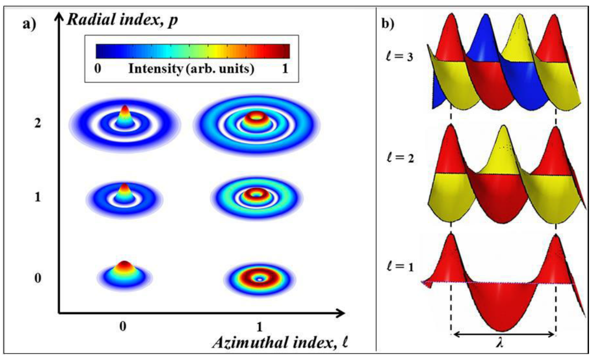



**Figure 1:** Intensity profiles and helical wavefront structures of Laguerre–Gaussian beams for varying radial and azimuthal indices.

Figure 1 shows Laguerre-Gaussian (LG) beams, where: a) a two-dimensional light intensity diagram shows LG beams with different mode indices. The rows and columns represent the mode indices in their azimuthal and radial forms, respectively. b) The helical wave front of LG beams is shown with an azimuthal index that varies from (m=1 to 3) while the radial index is kept at zero (n=0) (Mourka, 2025). The effect of the azimuth index (m) (or topological loading) on the radiation emission rate (SER) in orbital angular momentum (OAM)-based optical communication systems. In optical division multiplexing (OAM) systems, information is encoded into packets with different values of (m) (i.e., each value of (m) represents an independent channel). Each phase of type (exp(jm∅) ) in equation (1) represents a distinct mode that can be used to transmit independent data. This results in greater capacity, as several modes of (m) can be used in parallel. The higher the value of (m), the greater the beam's sensitivity to angular disturbances and, consequently, the higher the SER. Therefore, to minimize SER, the study selects the lowest possible value of (m).

**Amplitude field model.**

$$u_{rt}\,(r_x,r_y,L)\;=u_{rfs}(r_x,r_y,L)\exp[\Psi(r_x,r_y,L)] \quad (3)$$

The received field, $u_{rt}$ $(r_x,r_y,L)$ of a beam propagating through atmospheric turbulence over a distance (L) can be expressed according to Rytov theory, as described by Cheng *et al.* (2009). Consequently, the analytical expression for the average intensity of a Laguerre beam propagating through a turbulent atmosphere

$$\Psi\,(r_x,r_y,L)=\Psi_r(r_x,r_y,L)+j\,\Psi_i(r_x,r_y,L) \quad (4)$$

is formulated by employing the extended Huygens–Fresnel diffraction integral in conjunction with the Rytov approximation, and by using the random phase screen with the Kolmogorov model are done to check the behavior of the beam in weak and strong turbulence. Here, $(r_x,r_y)$ denote the transverse coordinates at the receiver plane, while $u_{rfs}$ $(r_x,r_y,L)$, represents the free-space propagating field. The influence of atmospheric turbulence is incorporated into the complex phase function $(r_x,r_y,L)$ , which itself can be further decomposed into distinct components.

The intensity fluctuations create scintillation on the receiver. By using equations. (3), and (4), the intensity on the received side $I_{rt}$ $(r_x,r_y,L)$ becomes

$$I_{rt}\,(r_x,r_y,L)=u_{rt}(r_x,r_y,L)u_{rt}^*(r_x,r_y,L)=u_{rfs}(r_x,r_y,L)u_{rfs}^*(r_x,r_y,L)\exp[2\Psi_r(r_x,r_y,L)]$$

$$=I_{rfs}(r_x,r_y,L)\exp[2\Psi_r(r_x,r_y,L)] \quad (5)$$

Where the intensity in open space is denoted by $I_{rfs}$ $(r_x,r_y,L)$

Equation (5) indicates that the fluctuations in the intensity function $I_{rt}$ $(r_x,r_y,L)$ arise exclusively from the real component of the complex phase term, which can be expressed as

$$\Psi_r\left(r_x, r_y, L\right) = 0.5 \ln\left[\frac{I_{rt}(r_x + r_y + L}{I_{rfs}(r_x + r_y + L}\right] \quad (6)$$

It is easier for a receiver to evaluate $I_{rfs}$ $(r_x,r_y,L)$ from the known analytical formulations for an optical communication system operating at a fixed distance of (L) and a known transmitted beam.

In the meantime, $I_{rt}$ $(r_x,r_y,L)$ is the received intensity at turbulence.

### The Modulation Technique

In the simulations, the constellation layout of the 4ASK modulation technique is depicted in Figure 2. The signal vectors' symbol endpoints are spaced uniformly by the optical power, with a maximum amplitude of one and a minimum of zero for the normalized vectors. This study selects a vortex Laguerre beam as the source beam, with its field expression provided in equation (1). The source beam will change when one of the 4ASK constellation's signal vectors modulates it:

$$u\,(s,\emptyset) = s_m * \left(\frac{s}{\propto_s}\right)^m \exp\left(-\frac{s^2}{\propto_s^2}\right) L_n^m\left(\frac{s^2}{\propto_s^2}\right)\exp(jm\emptyset) \quad (7)$$

Where:

$s_m=\{s_1,s_2,s_3,s_4\}=\{0.2^{0.5},0.5^{0.5},0.8^{0.5},1\}$

In this case, the separation between the terminal points of the signal vectors (symbols) is adjusted to be uniform with respect to optical power. The minimum normalized signal vector corresponds to an amplitude level of,$0.2^{0.5}$ whereas the maximum signal vector is assigned an amplitude level of unity. It should be noted that this

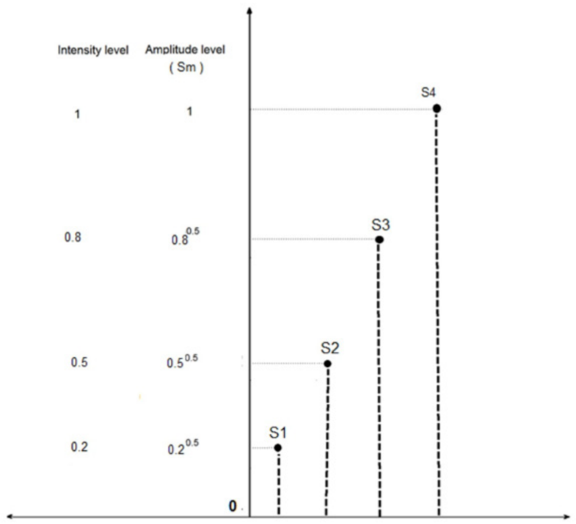



**Figure 2:** A plane source for a 4ASK signal vector constellation

unity level also represents the peak transmitted power under amplitude shift keying (ASK) modulation, and the condition is fulfilled when

$s_m=s_{m4}=1$.

### Configuration of random phase screen parameters

In constructing a random phase screen, the primary considerations are the grid interval and the number of sampling points (Ng) assigned to each rectangular dimension of the transverse plane at both the source and the receiver. These two factors govern the spatial resolution achievable in the receiver plane and establish the observation constraints associated with the transmitter plane (Pan *et al.*, 2019). The diffractive beam characteristics, the propagation medium's turbulence-induced spreading, and the source beam profile all influence the choice of grid spacing and number of grid factors. This operation also establishes the quantity of intermediate phase screen plates to be positioned between the transmitter and the receiver; for further details regarding these calculations and limitations (Plick & Krenn, 2015; Belmonte, 2000; Johnston & Lane, 2000). In applications involving RPS's s, the usual practice is to represent Ng as a power of 2.

Ng in the case of the study is assigned a value of 512, and the wavelength is 1550 nm. This implies that a high Ng value is required in order to achieve results that are very similar to those that the theory predicts due to its meaning of the number of runs or repetitions of the simulation. However, augmenting Ng also renders the calculations more time-consuming and intensive, as additional runs take more time and processing capabilities. The experiment has shown that Ng=500 is a good option, which corresponds to the values presented in the literature (Chimitt *et al.*, 2022). The quantitative results of this paper were determined based on the determination of ($\propto_s$=1 cm) and consideration of the source and propagation conditions, which resulted in the adoption of an appropriate RPS model. These steps are taken to make sure that the RPS arrangement would be able to resist extreme turbulence conditions in order to guarantee the reliability of the results.

In this case, the 4-ASK signals are sent across a distance of 3 km with a wavelength of 1550 nm. The level of turbulence of the air is represented by the structure constant (SC), which changes between $10^{-15}$ $m^{2/3}$ to $10^{-13}$ $m^{-2/3}$. According to these numeric values, it can be easily proved that the given system can effectively perform even when dealing with the strong turbulence conditions in the air, and meet the requirements (Zheng *et al.*, 2018). A stability switching (SSC) integration diversity scheme on a fading channel is presented. An evaluation of the results of the respective channel parameters is then plotted, and the influence of the fading intensity is monitored (Pranitha & Anjaneyulu,2019). The values of the bit error rate (BER) of the signal-to-noise ratio (SNR) of the different scattering effects in non-line-of-sight propagation of the UWAC system were determined (Eyadeh and Al-Ta'ani, 2019).

## RESULTS AND DISCUSSIONS

In this section, the findings derived using this study will be discussed, compared, and contrasted with the other studies.

### Results of Source Plane Intensities of Vortex Laguerre Beams

This section describes the relationship between the values of the Laguerre polynomial and the intensity pattern of a free-space optical beam, when the values of the Laguerre polynomials are changed. The radial parameter n, in this work, was held as a constant; the azimuthal parameter m was altered individually, with the values being (0, 1, 2, and 3). The following figures show the effect of various combinations of n and m on the pattern of the intensity of the beam. These variations of intensity make us comprehend how the symbol error rate (SER) varies in various situations; when n = 0 and m = 0, 1, 2, 3, and when n = 3 and m = 0, 1, 2, 3. On the other side, when (m=0) with (n=0,1,2, and 3) and (m=3) with (n=0,1,2, and 3), at the end (n=0,1,2, and 3) with (m=0,1,2, and 3). The view of transmitter plane intensities of Vortex Laguerre Beam with chosen values of Laguerre polynomial parameters can be observed in Figures 3, 4, 5, and 6, respectively. However, it can be seen clearly in Figures 4, 5, 6, and 7 that the near values for intensity against the axis are, in the case of a constant value, with a variety of values of (m) parameters and vice versa. However, when (n=3, m=1), (n=2, m=1), (n=1, m=0), and (n=0, m=0). It is observed that the values of source intensities against the axis are spaced out clearly, and these give good results for SER.

Li *et al.* (2024) examined the spiral-spectrum evolution of Laguerre-Gaussian beams through weak-to-strong Kolmogorov turbulence and were able to reveal that higher-order (n) radial beams spread energy over a wide range of spiral harmonics. This might lead to the peak intensity reduction; however, the spatial diversity increase, which is the mechanism that resists the localized scintillation and thus is responsible for lowering the symbol error probability in amplitude-keyed systems. Their spectral analysis has shown that the redistribution of modal weight by adjustment of the radial index increases the detection probability by a single central lobe in the presence of scintillation. Dong *et al.* (2022) found that when LG beams are partially coherent, and the correct mode choice is made, the scintillation index and beam wander are reduced as a result of the partial coherence and multiple mode integrations by higher n values, as demonstrated by independent numerical and experimental analysis of partially coherent LG beams. Therefore, a higher n can have a similar effect to a diversity mechanism, flattening the peaks and valleys of intensities at the receiver and reducing SER of ASK signals.

Moreover, the dependence on the mode shape of turbulence resilience is observed through comparisons of variations of LG modes (standard and elegant LG), in which the variations of LG mode shape influence turbulence resilience. Wei *et al.* (2020) found that the elegant LG formulations have greater resistance to turbulence-induced spreading in a wide range of parameter regimes than the standard LG, indicating that the integers (n, m) are not the only important quantities to optimize when it comes to SER. These results are consistent with the fact that (n=3, m=0) provides greater SER; it is the combination of an extended radial structure and low topological charge that stabilizes the received amplitude to perturbations by turbulent phases. These studies suggest two design principles of SER-sensitive links: (1) low azimuthal charge (m 0) is preferred to avoid being amplified by angular sensitivity and (2) higher radial index or modified LG families (or partially coherent variants) are chosen, which can help to redistribute energy spatially and suppress coherent scintillation effects which are also consistent with the simulation results (Li *et al.*, 2024).

### SER Results

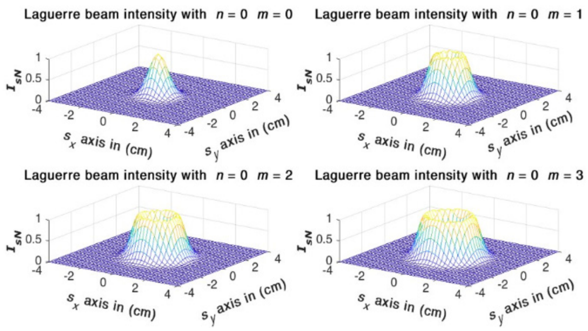



**Figure 3:** A three-dimensional view of source plane intensities of Vortex Laguerre Beam with n=0, and m=0, 1, 2, and 3.

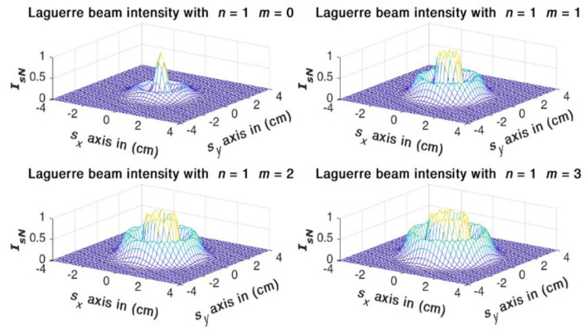



**Figure 4:** Three-dimensional representation of the source-plane intensity distribution for a Vortex Laguerre Beam with (n=1) and (m=0, 1, 2) and (3).

The symbol error rate (SER) is a measure that determines the number of errors that occur when receiving symbols compared to the number of symbols transmitted through the channel. Therefore, it is an important factor in evaluating system performance. This article focuses on the SER and seeks to reduce it to achieve optimal optical communication system performance. This section presents the outcomes of SER evaluations. As evidenced

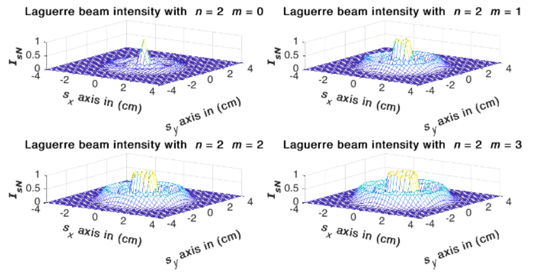



**Figure 5:** Three-dimensional illustration of the source-plane intensity distribution for a Vortex Laguerre Beam with (n=2) and (m=0, 1, 2, and 3).

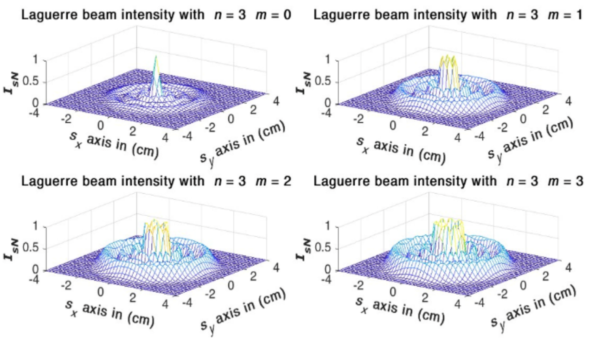



**Figure 6:** Three-dimensional illustration of the source-plane intensity distribution for a Vortex Laguerre Beam with (n=3) and (m=0, 1, 2,) and (3).

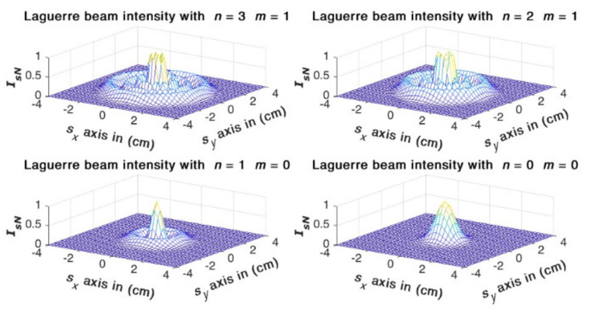



**Figure 7:** Three-dimensional illustration of the source-plane intensity distributions for Vortex Laguerre Beams with parameter sets (n=3, m=1), (n=2, m=1), (n=1, m=0), and (n=0, m=0).

from Fig. 7 to Fig. 11, the discrepancies in SC values from, $C_n^2=10^{-13}$- $10^{-15}$ $m^{-2/3}$ , in the unstable environment where the 4ASK signals have been relayed, significantly affects SER. In these figures, it is found that the graphs of SER are plotted against the structure constant ($C_n^2$), where a difference in values of $C_n^2$ , it was created. After that, it became clear that SER would spike sharply during periods of intense disturbance. It is evident that under the given conditions (n=0, m=0, and n=0, m=1), the behavior of SER is more favorable than under the conditions (n=0, m=2, and n=0, m=3); this is illustrated in Figure 8. In the same way, if the value of SC were to change, as shown in Figure 9, SER would rise rapidly once again.

In addition, for setting Laguerre polynomial parameters for the following (n=1, m=3, and n=1, m=2), but the behavior of SER is better in cases of (n=1, m=0, and n=1, m=1), and in the same way in Figures 10 and 11, respectively. Finally, it appears from Figure 12 that when Laguerre polynomial parameters are set in the following SER (n=3 with m=0), (n=2 with m=0), (n=1 with m=0), and (n=0 with m=0), the SER will be good, especially when polynomial parameters are adjusted to (n=3 with m=0) and (n=2 with m=0). Because larger (m) values cause the beam to be more sensitive to angular movements, thus increasing the radiation emission rate, and at the same time resulting in a higher symbol error rate (SER). The lowest symbol error rate (SER) that can be achieved under turbulent conditions is obtained by choosing the smallest (m) value, i.e., (m = 0). On the other hand, to obtain a low symbol error rate (SER), a high refractive index (n) is chosen for the Laguerre-Gaussian vortex beam, corresponding to the highest value obtained in these results (n=3). This, in turn, improves the performance of the optical transmission system in a turbulent environment.

The relationship between mode purity, topological charge, and error performance is supported by several recent analyses of BER/SER. Yang *et al.* (2022) evaluated the bit-error rates of Laguerre–Gaussian channels in absorbing and turbulent underwater links. They found that higher-order OAM modes produced larger BER under the same SNR conditions due to increased crosstalk and mode distortion. Although their study focused on underwater channels, the same mode-mixing mechanism applies to atmospheric turbulence, supporting the need to minimize m in ASK-based systems. Furthermore, previous studies such as Li *et al.* (2024), which explicitly considered spiral-spectrum transfer modeling, confirmed that both spectral widening and cross-mode energy transfer are directly proportional to turbulence strength. A source design that purposefully directs power to radial rings (higher n) and at the same time keeps the azimuthal index low would result in a scattering of energy to adjacent angular modes. Thus, the probability of symbol misclassification at the receiver is reduced. These theoretical and numerical results are in agreement with the simulation outcomes and indicate that the (n=3, m=0) configuration is in line with wider trends. The results of this study are compared with previous studies that used a Laguerre-based polynomial to obtain an optimal signal receiver based on the symbol error probability. The present study is further compared to other studies, including Liu *et al.* (2025), who introduced a mathematical model to shape signals in an optical system by the use of Laguerre functions. Their study addresses the impact of Laguerre expansions in the enhancement of the symbol error rate with the influence of advanced mathematics, and the result is the enhancement of optical communication systems. It is compared to the study carried out by Fareed *et al.* (2023), which presents a mathematical framework that is rigorous enough, showing how Laguerre functions could achieve better results in fiber-optic communication systems by reducing the error rate of the symbols. They conducted a profound analytical study of the use of Laguerre series in error reduction in optical fiber systems. The study presents an alternative view on how to enhance SER through readjusting the LG beam parameters instead of using sophisticated corrective mechanisms, which establishes a new and straightforward road leading to the creation of more efficient and cheaper technologies in optical communications. This point of view is highly supported by the visual evidence in Figures 13-16, which demonstrates that a proper choice of the parameters of the Laguerre polynomials can decrease

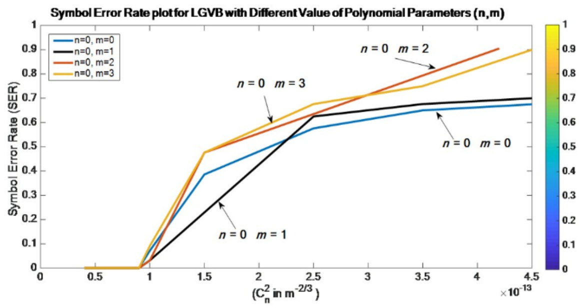



**Figure 8:** The effectiveness of changing of number of polynomials of LGVB on Symbol Error Rate SER with n=0, and m=0, 1, 2, and 3.

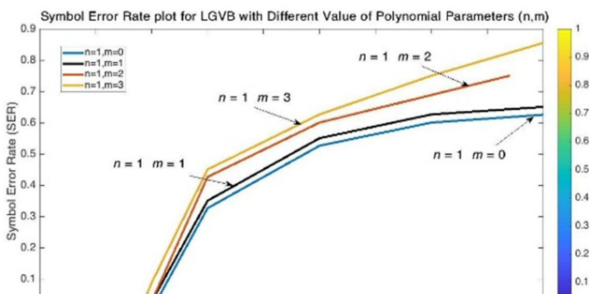



**Figure 9:** The effectiveness of changing of number of polynomials of LGVB on SER with n=1, and m=0, 1, 2, and 3.

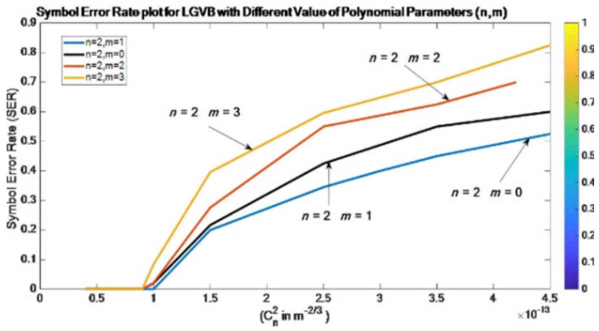



**Figure 10:** The effectiveness of changing of number of polynomials of LGVB on SER with n=2, and m=0, 1, 2, and 3.

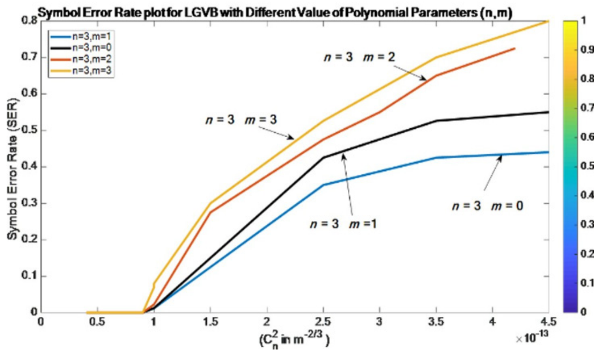



**Figure 11:** The effectiveness of changing of number of polynomials of LGVB on SER with n=3, and m=0, 1, 2, and 3.

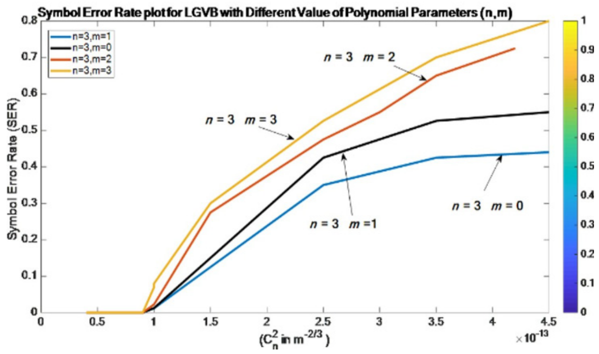



**Figure 12:** The effectiveness of changing of number of polynomials of LGVB on SER with n=0, and m=0, 1, 2, and 3, into moderate and high turbulence.

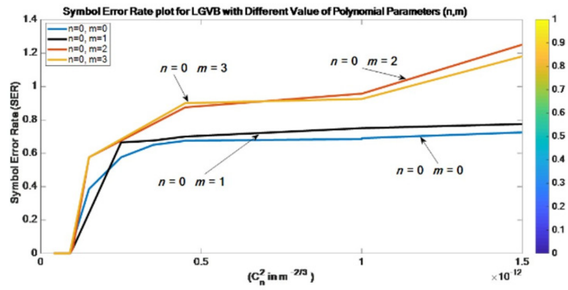



**Figure 13:** The effectiveness of changing of number of polynomials of LGVB on SER with n=1, and m=0, 1, 2, and 3, in moderate and high turbulence.

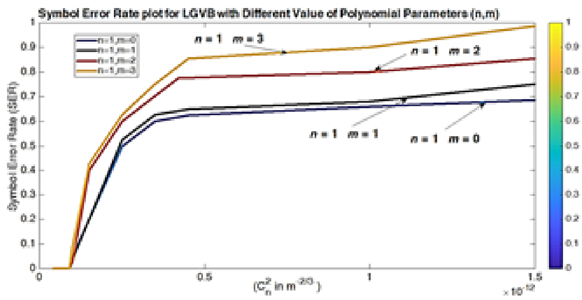



**Figure 14:** The effectiveness of changing of number of polynomials of LGVB on SER with n=1, and m=0, 1, 2, and 3, in moderate and high turbulence.

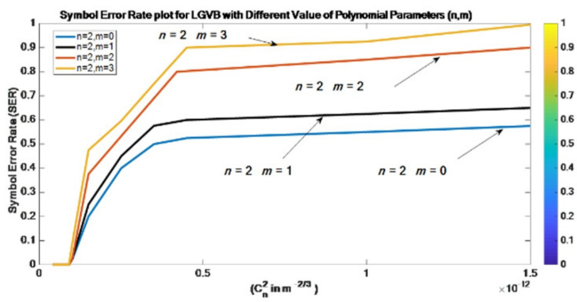



**Figure 15:** The effectiveness of changing of number of polynomials of LGVB on SER with n=1, and m=0, 1, 2, and 3, in moderate and high turbulence.

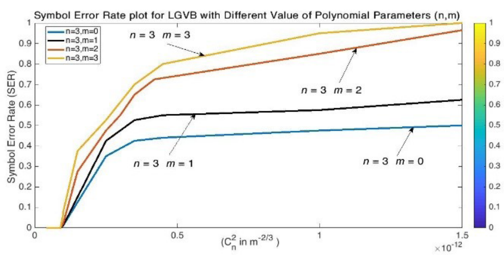



**Figure 14:** The effectiveness of changing of number of polynomials of LGVB on SER with n=2, and m=0, 1, 2, and 3, in moderate and high turbulence.

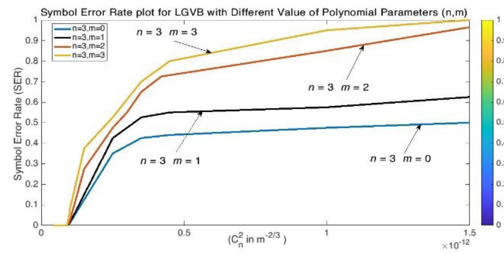



**Figure 16:** The effectiveness of changing of number of polynomials of LGVB on SER with n=3, and m=0, 1, 2, and 3, in moderate and high turbulence.

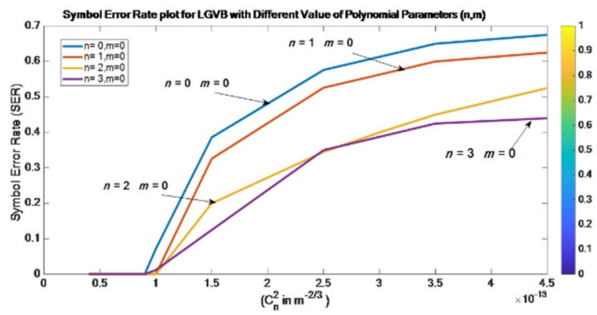



**Figure 17:** The effectiveness of changing of number of polynomials of LGVB on SER with n=3, and m=0, 1, 2, and 3, in moderate and high turbulence.

SER directly without complicated optical equipment.

## CONCLUSION

The symbol error rate (SER) under atmospheric turbulence was investigated using a simulation study that systematically varied the refractive index structure constant in the range Cn2 = 10-15 to 10-12 m-2/3, which is weak, moderate, and strong turbulence. Various Laguerre-Gaussian vortex beam (LGVB) configurations have been studied by changing the radial index n =0-3 and azimuthal index m=0-3. The findings indicate that the alterations of the parameters of polynomials have a significant impact on the performance of SER and the effectiveness of the beam intensity distribution. Most configurations experience a sharp increase in SER as the strength of the turbulence increases. However, beams that have a more significant radial index and a less significant azimuthal index are more robust. Specifically, it is found that the configuration (n = 3, m = 0) always produced the lowest SER at the moderate and strong turbulence conditions, and it outperformed the other parameter combinations. In contrast to previous research that is predominantly based on adaptive optics or on modulation schemes, the work also emphasizes the idea of beam-parameter optimization as an efficient and simple approach. The results indicate that the (n = 3, m = 0) LGVB is a good candidate in turbulence-resilient free-space optical and remote sensing operation.

## REFERENCES


Abdul, A. (2025). Predicting WSN packet loss using machine learning: Applications in solid surroundings. *Edelweiss Applied Science and Technology, 9*(2), 289–302. https://ideas.repec.org/a/ajp/edwast/v9y2025i2p289-302id4466.html

Alam, N., Wani, S. A., Khan, W. A., Fakhredine Gassem, & Anas Altaleb. (2024). Exploring Properties and Applications of Laguerre Special Polynomials Involving the Δh Form. *Symmetry, 16*(9), 1154–1154. https://doi.org/10.3390/sym16091154

Belmonte, A. (2000). Feasibility study for the simulation of beam propagation: consideration of coherent lidar performance. *Applied Optics, 39*(30), 5426. https://doi.org/10.1364/ao.39.005426

Cheng, W., Haus, J. W., & Zhan, Q. (2009). Propagation of scalar and vector vortex beams through turbulent atmosphere. SPIE Proceedings, 7200, 720004. https://doi.org/10.1117/12.809138

Chimitt, N., Zhang, X., Mao, Z., & Chan, S. H. (2022). Real-time dense field phase-to-space simulation of imaging through atmospheric turbulence. *IEEE Transactions on Computational Imaging, 8*, 1159-1169. https://arxiv.org/pdf/2210.06713

Dong, K., Cheng, M., Lavery, M. P., Geng, S., Wang, P., & Guo, L. (2022). Scattering of partially coherent vortex beam by rough surface in atmospheric turbulence. *Optics Express, 30*(3), 4165-4178. https://doi.org/10.1364/OE.444155

Doohwan, L., Yasunori, Y., Hirofumi, S., & Hiroyuki, S. (2022). Experimental Evaluation of High-capacity Wireless Transmission Using Orbital Angular Momentum Multiplexing Technology. *NTT Technical Review, 20*(3), 36-45. https://doi.org/10.53829/ntr202203ra1

Eyadeh, A. A., & Al-Ta'ani, M. N. (2019). Performance Study of Wireless Systems with Switch and Stay Combining Diversity over α-η-μFading Channels. *Engineering, Technology & Applied Science Research, 9*(6).

Eyyuboğlu, H. T., Y. Baykal, & A. Falits. (2011). Scintillation behavior of Laguerre Gaussian beams in strong turbulence. *Applied Physics B, 104*(4), 1001–1006. https://doi.org/10.1007/s00340-011-4588-y

Fareed, A., Ghazi, A., Aras Al-Dawoodi, Aljunid, S. A., Syed, Rashidi, M., Amphawan, A., Fakhrudeen, A. M., & Ibrahim. (2020). Comparison of Laguerre-Gaussian, Hermite–Gaussian and linearly polarized modes in SDM over FMF with electrical nonlinear equalizer. *AIP Conference Proceedings, 2203*, 020045–020045. https://doi.org/10.1063/1.5142137

Gregory, M., Troendle, D., G. Muehlnikel, Heine, F., Meyer, R., M. Lutzer, & R. Czichy. (2013). Three years coherent space to ground links: performance results and outlook for the optical ground station equipped with adaptive optics. *Proceedings of SPIE, the International Society for Optical Engineering/Proceedings of SPIE, 8610,* 861004–861004. https://doi.org/10.1117/12.2022253

Hricha, Z., Lazrek, M., Halba, M. E., & Belafhal, A. (2022). Effect of a turbulent atmosphere on the propagation properties of partially coherent vortex cosine-hyperbolic-Gaussian beams. *Optical and Quantum Electronics, 54*(11). https://doi.org/10.1007/s11082-022-04064-1

Huang, K., Xu, Y., & Li, Y. (2023). Average Intensity of a Laguerre—Gaussian Vector Vortex Beam through Inhomogeneous Atmospheric Turbulence. *Photonics, 10*(11), 1189–1189. https://doi.org/10.3390/photonics10111189

Jiang, R., Wang, K., Tang, X., & Wang, X. (2023). Investigation of Oceanic Turbulence Random Phase Screen Generation Methods for UWOC. *Photonics, 10*(7), 832. https://doi.org/10.3390/photonics10070832

Johnston, R. A., & Lane, R. G. (2000). Modeling scintillation from an aperiodic Kolmogorov phase screen. *Applied Optics, 39*(26), 4761–4761. https://doi.org/10.1364/ao.39.004761

Li, M., & Cvijetic, M. (2015). Coherent free space optics communications over the maritime atmosphere with use of adaptive optics for beam wavefront correction. *Applied optics, 54*(6), 1453-1462. https://doi.org/10.1364/ao.54.001453

Li, Y., Zhang, Z., Li, R., Xu, D., Zhang, H., Cai, Y., & Zeng, J. (2024). The Spiral Spectrum of a Laguerre–Gaussian Beam Carrying the Cross-Phase Propagating in Weak-to-Strong Atmospheric Turbulence. *Photonics, 11*(2), 148. https://doi.org/10.3390/photonics11020148

Liu, M., Yao, L., Gao, Y., Zhang, H., Yuan, Y., Guo, B., & Shi, H. (2025). Optical Communication Performance of Cylindrical Vector Partially Coherent Laguerre–Gaussian Beams in Atmospheric Turbulence. *Photonics, 12*(9), 883. https://doi.org/10.3390/photonics12090883

Mackey, R., & Dainty, C. (2005). Wavefront sensing and adaptive optics in strong turbulence. *SPIE Proceedings, 5827*, 23. https://doi.org/10.1117/12.605071

Mourka, A. (2025). Probing the modal characteristics of novel beam shapes. St-Andrews.ac.uk. https://hdl.handle.net/10023/4287

Nossir, N., Dalil-Essakali, L. & Belafhal, A. Behavior of the central intensity of generalized humbert-gaussian beams against the atmospheric turbulence. *Opt Quant Electron 53,* 665 (2021). https://doi.org/10.1007/s11082-021-03316-w

Pan, S., Wang, L., Wang, W., & Zhao, S. (2019). An Effective Way for Simulating Oceanic Turbulence Channel on the Beam Carrying Orbital Angular Momentum. *Scientific Reports, 9*(1). https://doi.org/10.1038/s41598-019-50465-w

Plick, W. N., & Krenn, M. (2015). Physical meaning of the radial index of Laguerre-Gauss beams. *Physical Review A, 92*(6). https://doi.org/10.1103/

physreva.92.063841

Pranitha, B., & Anjaneyulu, L. (2019). Performance Evaluation of a MIMO based Underwater Communication System under Fading Conditions. Engineering, Technology & Applied Science Research, 9(6).

Schwartz, N. H., Védrenne, N., Michau, V., Velluet, M.-T., & Chazallet, F. (2009). Mitigation of atmospheric effects by adaptive optics for free-space optical communications. *SPIE Proceedings, 7200,* 72000J. https://doi.org/10.1117/12.808142

Sharma, S. (2012). A simplified free-space adaptive optics system against atmospheric turbulence. *International Journal of Electronics, 99*(3), 417-436. https://doi.org/10.1080/00207217.2011.629222

Stotts, L. B., Kolodzy, P., Pike, A., Graves, B., Dougherty, D., & Douglass, J. (2010). Free-space optical communications link budget estimation. *Applied Optics, 49*(28), 5333–5333. https://doi.org/10.1364/ao.49.005333

Wang, Y., Xu, H., Li, D., Wang, R., Jin, C., Yin, X., Gao, S., Mu, Q., Xuan, L., & Cao, Z. (2018). Performance analysis of an adaptive optics system for free-space optics communication through atmospheric turbulence. *Scientific Reports, 8*(1), 1124–1124. https://doi.org/10.1038/s41598-018-19559

Wei, D., Li, S., Zeng, J., Zhu, X., Chen, T., Cai, Y., & Yu, J. (2020). Comparative Study of Spiral Spectrum of Elegant and Standard Laguerre–Gaussian Beams in Atmospheric Turbulence. *Journal of Russian Laser Research, 41*(4), 364–372. https://doi.org/10.1007/s10946-020-09887-5

Yang, H., Yan, Q., Wang, P., Hu, L., & Zhang, Y. (2022). Bit-error rate and average capacity of an absorbent and turbulent underwater wireless communication link with perfect Laguerre-Gauss beam. *Optics Express, 30*(6), 9053–9064. https://doi.org/10.1364/OE.451981

Zeng, J., Liu, X., Zhao, C., Wang, F., Gbur, G., & Cai, Y. (2019). Spiral spectrum of a Laguerre-Gaussian beam propagating in anisotropic non-Kolmogorov turbulent atmosphere along horizontal path. *Optics Express, 27*(18), 25342–25342. https://doi.org/10.1364/oe.27.025342

Zheng, D., Li, Y., Zhou, H., Bian, Y., Yang, C., Li, W., Qiu, J., Guo, H., Hong, X., Zuo, Y., Giles, I. P., Tong, W., & Wu, J. (2018). Performance enhancement of free-space optical communications under atmospheric turbulence using modes diversity coherent receipt. *Optics Express, 26*(22), 28879–28879. https://doi.org/10.1364/oe.26.028879