American Journal of Innovation in Science and Engineering (AJISE)

ISSN: 2158-7205 (ONLINE)

VOLUME 5 ISSUE 2 (2026)

PUBLISHED BY
E-PALLI PUBLISHERS, DELAWARE, USA

American Journal of Innovation in Science and Engineering (AJISE)
Volume 5 Issue 2, Year 2026
ISSN: 2158-7205 (Online)
DOI: https://doi.org/10.54536/ajise.v5i2.7091
https://journals.e-palli.com/home/index.php/ajise

Integrated Inventory Model for Multi Item under Price - Sensitive Demand with Controllable Lead Time and Greenhouse Gas Emission Effect on Production and Transportation

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Article Information

Received: November 12, 2025
Accepted: February 18, 2026
Published: May 11, 2026

Keywords

Greenhouse Gas Effect, Integrated Inventory Model, Lead Time, Multi Items, Price Sensitive Demand

ABSTRACT

In today's era of rapid human advancement, environmental concerns have become just as important as economic and industrial growth. This study examines how greenhouse gas (GHG) emissions affect production and transportation in a supply chain model in which a single vendor supplies multiple items to several buyers. The buyers' demand is normally distributed, and delivery lead times can be shortened by incurring crash costs. Importantly, the model also incorporates transportation and GHG-related costs during both production and delivery stages. Additionally, the model considers price-sensitive demand, recognizing that fluctuations in price can influence buyer behavior and overall demand levels. The main goal is to minimize the overall expected cost while determining key decision variables, such as each buyer's lead time, the vendor's order quantities, and shipment frequency. A numerical example is used to illustrate the model, and a sensitivity analysis explores how changes in key parameters affect the optimal supply chain decisions.

INTRODUCTION

Inventory management is similar to a balancing act, where the goal is to maintain enough items in stock to fulfill orders, without accumulating excess that would lead to high costs. Inventory models help companies predict how much they will need, figure out when to order more, and avoid problems like running out of stock or having too much. In a supply chain, raw materials are delivered from the supplier to the manufacturer, and then finished products are transferred from the manufacturer to the buyer. The main goal of every supply chain is to minimize the total cost and maximize profits.

In today's globalized business environment, it is essential to integrate decision-making processes across production, inventory, and distribution to ensure efficiency. Coordinating these decisions across the supply chain and reducing lead times are recognized as effective strategies for achieving an efficient and responsive supply chain. Banerjee (1986) developed a joint economic lot size model in which the vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions. Later, Goyal (1988) modified Banerjee's model and provided a lower or equal joint total relevant cost. Later, Lu (1995) proposed a model in which shipments are allowed to be taken during the production period as well.

At the same time, customer behavior has changed significantly. Demand for products is no longer constant and is often sensitive to price. When prices increase, demand usually decreases, and when prices decrease, demand tends to rise. This relationship, known as price sensitive demand, must be incorporated into inventory decisions because pricing directly influences order quantities, production planning, and profit levels. Ignoring price sensitivity may lead to incorrect estimation of demand and poor inventory policies.

Another major concern in recent years is the impact of industrial activities on the environment, especially through greenhouse gas (GHG) emissions. Production processes and transportation activities are two major sources of these emissions in a supply chain. Governments and environmental agencies are imposing strict regulations and carbon policies to control emissions, which directly affect operational costs. As a result, companies must now consider environmental factors along with economic objectives while designing inventory systems.

Furthermore, many practical situations involve handling multiple items rather than a single product. Managing multiple items simultaneously is more complex because each item may have different demand patterns, costs, and emission impacts. An integrated inventory model for multiple items helps in achieving better coordination, optimal resource utilization, and improved overall performance of the supply chain.

Considering all these factors, there is a growing need to develop integrated inventory models that incorporate multi-item management, price sensitive demand, controllable lead time, and the effect of greenhouse gas emissions from both production and transportation. Such models can help organizations make more realistic and sustainable decisions by balancing economic performance with environmental responsibility.

LITERATURE REVIEW

Most inventory research focuses on the single-vendor, single-buyer inventory models. However, in today's globalized world, not all supply chain situations are like

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this. Many industries involve multiple parties in supply chain activities. Therefore, exploring research on single-vendor, multi-buyer model is crucial. This can provide us with better ideas on how to run an efficient supply chain in more realistic situations. Banerjee *et al.* (1994) developed a model in which orders are placed at every fixed time interval. To meet these time demands, the vendor uses a production cycle that is a multiple of the fixed time interval. Sarmah *et al.* (2008) looked into a situation where a company supplies its products to many buyers all at once, following a set schedule for restocking. By doing this, the company can use the same transportation for all deliveries, which helps save money. Jha *et al.* (2013) applied service level constraints (SLC) in a single-vendor multi-buyer scenario for a single item with a continuous review policy. Aritonang *et al.* (2021) modified the idea from a single-item concept to a multiple-item concept consisting of several buyers and a single vendor, which is more realistic.

Traditional inventory models often focus on managing a single product. However, such single-item scenarios are rarely encountered in real-world operations. In actuality, businesses, retailers, and organizations typically handle a variety of products and maintain a diverse inventory within their storage facilities. Classical multi-item inventory models that operate under resource limitations—such as storage space, financial budgets, average inventory levels, and delivery frequency—are extensively covered in standard literature. For example, Ben-Daya and Raouf (1993) developed a probabilistic demand-based inventory model for multiple products, considering space and budget limitations. More recently, Taleizadeh *et al.* (2011) explored a complex supply chain setup involving multiple buyers and vendors, where each buyer faced purchase capacity constraints and vendors had limited storage capabilities. Ghosh *et al.* (2015) analyzed a multi-product scenario with budget and space constraints, where demand was influenced by both stock levels and product deterioration. Pasandideh *et al.* (2017) addressed delayed payments in a multi-product model and applied genetic algorithms for optimization. Similarly, Dutta *et al.* (2017) used genetic algorithms to solve a multi-objective fuzzy stochastic inventory problem. Singh *et al.* (2017) examined an economic production quantity (EPQ) model for products that deteriorate over time, incorporating a demand rate that increases exponentially with time and is linked to the production rate. Later, Ogunleye *et al.* (2022) developed a non-pre-emptive integer nonlinear goal programming model for a multi-item inventory system to minimize inventory costs and maintain optimal stock levels, demonstrated through a case study of a car retail centre in Lagos State.

When dealing with uncertain demand, lead time becomes a crucial factor in managing inventories and distribution between buyers and vendors. Shortening lead time can enhance responsiveness to market fluctuations Liao *et al.* (1991) devised an inventory model centered on lead time, and normally distributed demand. Their model has inspired numerous other researchers, such as Pan *et al.* (2002), Hoque *et al.* (2006), and Lin *et al.* (2011), who integrated the concept of lead time reduction into various models. Lead time can be reduced by crashing costs. Li *et al.* (2011) considered controllable lead time in their model. Jha *et al.* (2013) proposed a model with controllable lead time, a continuous review policy, and service level constraints for each buyer. Subsequently, Hoque (2013) developed a model assuming equal and unequal batches of shipments with an exponential distribution of lead time Vijayashree *et al.* (2016) also developed a model with the possibility of reducing lead time and using exponential function to estimate crashing cost. Giri *et al.* (2020) introduced another model with stochastic lead time and price and quantity-dependent demand. Later, Giri *et al.* (2023) modified their own supply chain model to incorporate price and green-sensitive demand under stochastic lead time, while Chowdhury *et al.* (2023) developed a smart inventory management system integrated with forecasting techniques to improve the efficient handling and monitoring of industrial assets.

Many researchers also utilize transportation factors in their supply chain models. Hoque *et al.* (2000) proposed a model wherein a fixed transportation cost is applied for each shipment. Later, Hsiao (2008) incorporated both transportation cost and transportation time into a single vendor single buyer inventory model. Hoque (2011) further refined the concept by incorporating vehicle capacity along with transportation cost and time. Jha *et al.* (2013) introduced the innovative concept of vehicle routing with multiple vehicles in a single vendor-multi buyer supply chain. Subsequently, numerous researchers (kurdhi *et al.* (2021), Castellano *et al.* (2019) have applied vehicle routing in their inventory models.

For further refinement of transportation factors, some researchers have developed models to assess the impact of greenhouse gas emissions (GHG). Introducing costs for greenhouse gas emissions is an effort to reduce environmental harm from human activities. Initially, Glock *et al.* (2015) introduced a model integrating GHG emissions with the vehicle routing problem. This model handles a supply chain where one buyer has different types of vehicles and picks up products from different suppliers using a milk run approach. Subsequently, Castellano *et al.* (2019) demonstrated the influence of GHG emissions on production and transportation within their single vendor multi buyer supply chain model. Later, many researchers adopted the concept of greenhouse gas emissions in supply chain models (Noh *et al.* (2019), Marchi *et al.* (2019), Adhikary *et al.* (2020),suef *et al.*(2023)).

In our supply chain model, which considers a single vendor and multiple buyers, we consider controllable lead time and the effects of greenhouse gas emissions on production and transportation. The total inventory cost,which serve as the supply chain performance indicator to be minimized in this model, encompasses various factors. These include the vendor's holding costs and setup cost, as well as the buyer's holding costs, purchasing costs, greenhouse gas emission costs, and transportation cost.

MATERIALS AND METHODS

Theoretical Framework

In our supply chain model, which considers a single vendor and multiple buyers, we consider controllable lead time and the effects of greenhouse gas emissions on production and transportation. The total inventory cost,which serve as the supply chain performance indicator to be minimized in this model, encompasses various factors. These include the vendor's holding costs and setup cost, as well as the buyer's holding costs, purchasing costs, greenhouse gas emission costs, and transportation cost.

Notations

To develop the mathematical model the following notations and assumptions are used:

Index

Index for products, $i=1,2,3\dots M$

Index for buyers, $j=1,2,3\dots N$

Decision Variable

m: Number of lots delivered from vendor to each buyer in production cycle

Q_{ij}: Order quantity of i^{th} product for j^{th} buyer, $Q=\sum_{i=1}^{M}\sum_{j=1}^{N} Q_{ij}$

L_{ij}: Length of lead time of i^{th} product for j^{th} buyer

Parameters

P_{ij}: Production rate of i^{th} product for j^{th} buyer P_{ij} ($P_{ij}>D_{ij}$)

D_{ij}: Demand rate of i^{th} product for j^{th} buyer

A_{vi}: Set cost per setup for product i

C_{vi} : Unit production cost for product i

Ab_{ij}: Ordering cost per order of i^{th} product for j^{th} buyer

Cb_{ij}: Unit purchase cost of i^{th} product for j^{th} buyer

Hb_{ij}: Holding cost rate per unit time of i^{th} product for j^{th} buyer

Q_{ij}: Order quantity of i^{th} product for j^{th} buyer

s_{ij}: Cost of placing an order of i^{th} product for j^{th} buyer

T_{ij}: Cost of transporting a batch from the manufacturer to the i^{th} product for j^{th} buyer

p_{ij}: Unit retail price of i^{th} product for j^{th} buyer

K: Number of routes

l_k: Length of route

C_1: Variable cost

f: Cost of operating a vehicle

a: Emissions function parameter

b: Emissions function parameter

c: Emissions function parameter (ton/quantity)

e_1: Emissions tax

β: Average fuel consumed

γ: Average amount of GHG emissions from fuel

N: Number of buyers

M: Number of products

Assumptions

The mathematical model is based on the following assumptions:

1. The vendor produces a multi items and meets the demand of multiple buyers.
2. The j^{th} buyer places order of quantity Q_{ij} and the vendor manufactures mQ_{ij} units with production rate $P_{ij}(P_{ij}>D_{ij})$ in one setup in one setup but ship in quantity over n times to meet the demand of all time buyers.
3. The lead time of i^{th} product and j^{th} buyer L_{ij}consist of m_{ij} mutually independent components. The r^{th} component of lead time of buyer i has a minimum duration a_{ijr}, a normal duration b_{ijr} and a crash cost per unit time C_{ijr} s.t. $C_{ij1}\le C_{ij2}\le\cdots\le C_{ijmij}$ ∀ i,j
4. The lead time components of each buyer are crashed one at a time starting with the least one at a time starting with the least crash cost $C_{i,r}$ ∀i component and so on.
5. Let$\sum_{r=1}^{mij} b_{ijr}=L_{ij0}$,∀r=1,2,…,$m_{ij}$ denote the maximum duration of lead time of buyer j, and L_{ijr} be the length of lead time for buyer i with components 1,2…,r crashed to their minimum duration, then $L_{ijr}=\sum_{J=1}^{mij} b_{ijr} +\sum_{r=1}^{mij} a_{ijr}$,r=1,2,…,m_{ij}. The lead time crashing cost $C(L_{ij})$ per cycle of the i^{th} buyer for a given $L_{ij}\in(L_{ijr},L_{ij(r-1)})$ is given by $C(L_{ij})=C_{ijr}(L_{ij(r-1)}-L_{ij})+\sum_{J=1}^{r-1} C_{ijJ}(L_{ij(J-1)}-L_{ij})$
6. The buyer's demand rate depends linearly on the selling price of the product demand rate of j^{th} buyer and i^{th} products is $D_{ij}(p_{ij})=\alpha_{ij}-\beta_{ij} p_{ij}$, where α_{ij} represents the basic consumer demand and β_{ij} is a positive integer s.t. $\alpha_{ij}>\beta_{ij} p_{ij}$ ∀ i=1,2,3…N
7. Shortages are not allowed.
8. Shipments from vendor to buyer are made using identical vehicles.
9. The two main factors in greenhouse gas emission are the production processes at vendor's end and the transportation of goods from vendor to buyers. Other sources for GHG emissions are not considered in this model.

The Mathematical Model

In this section, we develop a comprehensive mathematical model, aimed to minimize the overall expected cost per unit time and determining the optimal order quantity, number of shipments, and lead time for each buyer. This joint total expected cost comprises the total cost of each buyer, the total cost of the vendor, transportation cost and greenhouse gas emissions cost. The vendor's total cost encompasses production and inventory holding costs. Meanwhile, each buyer's total cost is a combination of ordering, holding, and crashing costs. Greenhouse gas emission costs are attributed to emissions from both production and transportation processes. Transportation costs are contingent upon vehicle routing costs, which include fixed and variable costs based on the distance traveled.

Total Expected Cost Per Unit Time for Vendor

Once the orders are placed by all the buyers, the production begins and the vendor produces and delivers the item simultaneously. The vendor produces mQ_{ij} quantity in one production cycle and each buyer receive it in m lots each of size Q_{ij}.

The vendor's total expected cost per unit time includes the setup cost and inventory holding cost. The setup cost per unit time is $A_{Vi} D_{ij} (p_{ij})/mQ_{(ij)}$ and vendor's average inventory is the difference of the vendor's accumulated inventory and buyer's accumulated inventory (figure1, figure2) therefore, inventory holding cost per unit time is $h_{vi} C_{vi} (Q_{ij}/2)[m(1-(D_{ij}(p_{ij}))/P_{ij})-1+(2D_{ij})/P_{ij}]$. Therefore, vendor's total expected cost per unit time of ith product for the jth buyer is

$$JTEC_v(Q_{ij},n) = \frac{A_{Vi}D_{ij}(p_{ij})}{mQ_{ij}} + h_{vi}C_{vi}\left(\frac{Q_{ij}}{2}\right)\left[n\left(1-\frac{D_{ij}(p_{ij})}{P_{ij}}\right)-1+\frac{2D_{ij}}{P_{ij}}\right] \quad (1)$$

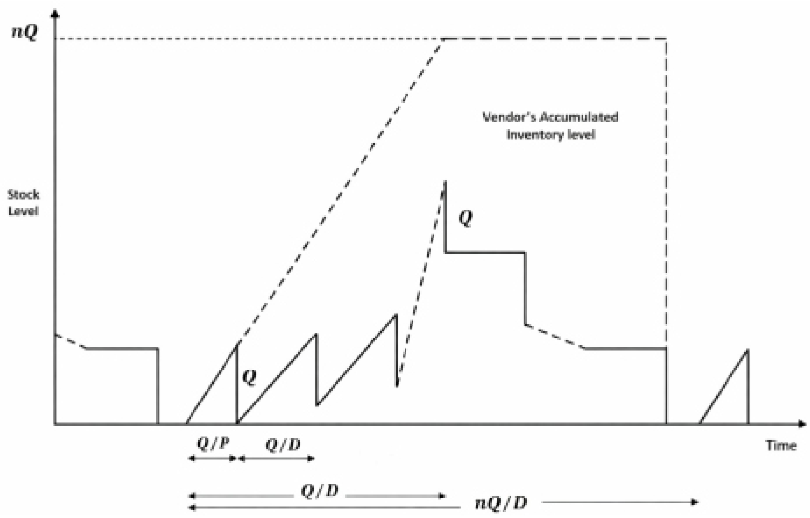

Figure 1: The inventory pattern for vendor

Total Expected Cost Per Unit Time for ith Buyer

The inventory pattern for the buyer is shown in figure 2. To formulate the total expected cost for the buyers, it's necessary to calculate the separate cost for each buyer. The buyer's total expected cost per unit time is combination of ordering cost, holding cost and crashing cost of each buyer. For the ith buyer ordering cost per unit time is $A_{bij} D_{ij} (p_{ij})/Q_{ij}$, inventory holding cost per unit time is $h_{bij} C_{bij} (Q_{ij}/2)$ and the lead time crashing cost per unit time $(D_{ij}/Q_{ij}) C_{ij} (L_{ij})$. Therefore the ith total expected cost per unit time is

$$TEC_{bi}(Q_{ij},n) = \frac{A_{bij}D_{ij}(p_{ij})}{Q_{ij}} + h_{bij}C_{bij}\left(\frac{Q_{ij}}{2}\right) + \left(\frac{D_{ij}}{Q_{ij}}\right)C_{ij}(L_{ij}). \quad (2)$$

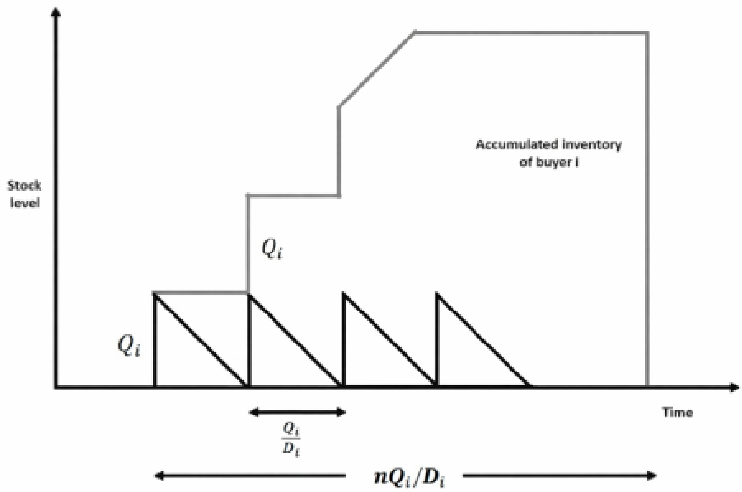

Figure 2: The inventory pattern for ith buyer

Total Expected Transportation Cost Per Unit Time

The vendor distributes products to buyers using vehicles of uniform capacity. Each vehicle operates along a designated route that may serve multiple buyers. The transportation cost associated with each delivery consists of two components: a fixed cost, incurred per dispatch regardless of the distance, and a variable cost, which is proportional to the total distance traveled by the vehicle along its route. Accordingly, the total expected transportation cost per unit time is determined by the combined effect of these two components.

$$TEC_T(Q_{ij}) = \left(\frac{D_{ij}}{Q_{ij}}\right)\left(Kf + C_1\sum_{g=1}^{K} l_g\right) \quad (3)$$

Total Expected GHG Emission Cost Per Unit Time

Two main sources of greenhouse gas emissions that we consider in this model are production and transportation. An emission cost is considered as a tax, paid for every ton of GHG emitted.

The amount of GHG emissions (in tons per unit quantity) from a production process is $f(P_{ij})=aP_{ij}^2-bP_{ij}+c$.

Total amount of GHG emission form production for demand D_{ij} is $E_P=D_{ij}(aP_{ij}^2-bP_{ij}+c)$, Where parameters a, b, and c are depend on production process.

Therefore, the GHG emissions cost per unit time due to production$=e_1 E_P$

The greenhouse gas emissions due to transportation are directly proportional to the amount of fuel consumed, thus the amount of total emissions (per unit time) from transportation is

$E_T=(D_{ij}/Q_{ij})\sum_{k=1}^{K}\beta\gamma l_k$.

Therefore, the GHG emissions cost per unit time due to transportation$=e_1 E_T$

So, total expected greenhouse gas emission (GHG) cost per unit time due to production and transportation

$$TEC_G(Q_{ij}) = e_1\left[D_{ij}(aP_{ij}^2 - bP_{ij} + c) + \left(\frac{D_{ij}}{Q_{ij}}\right)\sum_{k=1}^{K}\beta\gamma l_k\right] \quad (4)$$

Joint Total Expected Cost

The joint total expected cost is the sum of total expected cost of vendor, total expected cost of buyer, total expected transportation cost and total expected emission cost. Therefore, joint total expected cost is

$$JTEC(Q_{ij}, m, L_{ij}) = \sum_{j=1}^{M}\sum_{i=1}^{N} TEC_v(Q_{ij}, m, L_{ij}) + \sum_{j=1}^{M}\sum_{i=1}^{N} TEC_{bi}(Q_{ij}, m, L_{ij}) + \sum_{j=1}^{M}\sum_{i=1}^{N} TEC_T(Q_{ij}, m, L_{ij}) + \sum_{j=1}^{M}\sum_{i=1}^{N} TEC_G(Q_{ij}, m, L_{ij})$$

$$JTEC(Q_{ij}, m, L_{ij}) = \sum_{j=1}^{M}\sum_{i=1}^{N}\Big[\frac{A_{vi}D_{ij}(p_{ij})}{mQ_{ij}} + h_{vi}C_{vi}\left(\frac{Q_{ij}}{2}\right)\left[m\left(1-\frac{D_{ij}(p_{ij})}{P_{ij}}\right) - 1 + \frac{2D_{ij}(p_{ij})}{P_{ij}}\right] + \frac{A_{bij}D_{ij}(p_{ij})}{Q_{ij}} + h_{bij}C_{bij}\left(\frac{Q_{ij}}{2}\right) + \left(\frac{D_{ij}}{Q_{ij}}\right)C_{ij}(L_{ij}) + \left(\frac{D_{ij}}{Q_{ij}}\right)\left(Kf + C_1\sum_{g=1}^{K} l_g\right) + e_1\left[D_{ij}(aP_{ij}^2 - bP_{ij} + c) + \left(\frac{D_{ij}}{Q_{ij}}\right)\sum_{k=1}^{K}\beta\gamma l_k\right]$$

(5)

Solution Technique

To determine the minimum total expected cost per unit time $JTEC(Q_{ij},m,L_{ij})$, we have to determine the optimal solutions from decision variable Q_{ij},m and L_{ij}. Firstly, for fixed m and L_{ij}. First order partial derivatives of $JTEC(Q_{ij},m,L_{ij})$ with respect to Q_{ij}

$$\left(\frac{\partial JTEC(Q_{ij},m,L_{ij})}{\partial Q_{ij}}\right) = -\left(\frac{A_{vij}D_{ij}(p_{ij})}{mQ_{ij}^2}\right) + \left(\frac{h_{vi}C_{vi}}{2}\right)\left[m\left(1-\frac{D_{ij}(p_{ij})}{P_{ij}}\right) - 1 + \frac{2D_{ij}(p_{ij})}{P_{ij}}\right] - \sum_{i=1}^{N}(A_{bij}D_{ij}(p_{ij})/Q_{ij}^2) + \sum_{i=1}^{N}\left(\frac{h_{bij}C_{bij}}{2}\right) - e_1 D_{ij}(p_{ij})/Q_{ij}^2)\sum_{k=1}^{K}\beta\gamma l_k - (D_{ij}/Q_{ij}^2)[fK + C_1\sum_{g=1}^{K} l_g] \quad (6)$$

Now for fixed n and $L_{ij} \in (L_{ijr}, L_{ijr-1}$ second order partial derivative of $JTEC(Q_{ij}, m, L_{ij})$ ∀ i,j with respect to Q_{ij} is

$$\left(\frac{\partial^2 JTEC(Q_{ij},m,L_{ij})}{\partial Q_{ij}^2}\right) = \left(\frac{2D_{ij}(p_{ij})}{Q_{ij}^3}\right)\left[\left(\frac{A_{vi}}{m}\right) + \sum_{i=1}^{N}(A_{bij}) + e_1\sum_{k=1}^{K}\beta\gamma l_k + (fK + C_1\sum_{g=1}^{K} l_g) > 0 \quad (7)\right.$$

(since all the parameters are positive)

Therefore, for fixed n and $L_{ij} \in (L_{ijr}, L_{ijr-1})$∀i and j, $JTEC(Q_{ij}, m, L_{ij})$ is convex in Q_{ij}

Similarly for fixed Q_{ij} and $L_{ij} \in (L_{ijr}, L_{ijr-1})$ first order partial derivative of $JTEC(Q_{ij}, m, L_{ij})$ with respect to m, we get

$$\left(\frac{\partial JTEC(Q_{ij},m,L_{ij})}{\partial m}\right) = -(A_{vi}D_{ij}(p_{ij})/m^2Q_{ij}) + (h_{vi}C_{vi}Q_{ij}/2)(1 - D_{ij}(p_{ij})/P_{ij}) \quad (8)$$

Now for fixed Q_{ij} and $L_{ij} \in (L_{ijr}, L_{ijr-1})$ second order partial derivative of $JTEC(Q_{ij}, m, L_{ij})$ with respect to m

$((\partial^2 JTEC(Q_{ij},m,L_{ij}))/\partial m^2)=(2A_{vi} D_{ij}(p_{ij})/m^3 Q_{ij})>0$ (9)

(since all the parameters are positive)

Therefore, for fixed Q_{ij}, and $L_{ij} \in (L_{ijr}, L_{ijr-1})$ ∀i,j, $JTEC(Q_{ij}, m, L_{ij})$ is convex in m

Similarly for fixed Q_{ij} and m first order partial derivative of $JTEC(Q_{ij}, m, L_{ij})$ with respect to $L_{ij} \in (L_{ijr}, L_{ijr-1})$ ∀i,j, we get

$\partial JTEC(Q_{ij},m,L_{ij})/(\partial L_{ij})=-((D_{ij}(p_{ij}))/Q_{ij})C_{ijr}$ (10)

Now the second order derivative of $JTEC(Q_{ij}, m, L_{ij})$ with respect to $L_{ij} \in (L_{ijr}, L_{ijr-1})$ ∀i,j, for fixed Q_{ij} and m

$((\partial^2 JTEC(Q,m,Li))/\partial L_i^2)=0$ (11)

Since $JTEC(Q_{ij}, m, L_{ij})$ is a linear function of L_{ij} so as L_{ij} increases $JTEC(Q_{ij}, m, L_{ij})$ also increases. Therefore, for minimum total cost will occur at starting point of the interval i.e. L_{ij} .

Now putting equation (6) to zero, we have

$$Q_{ij} = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}[D_{ij}(p_{ij})(\frac{A_{vi}}{m} + \sum_{i=1}^{N}(A_{bij}) + C_{ij}(L_{ij}) + e_1(\sum_{k=1}^{K}\beta\gamma l_k) + (fK + C_1\sum_{k=1}^{K} l_k)]}{\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{h_{vi}C_{vi}}{2}\right)\left[m\left(1-\frac{D_{ij}(p_{ij})}{P_{ij}}\right) - 1 + \frac{2D_{ij}(p_{ij})}{P_{ij}}\right] + \sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{h_{bij}C_{bij}}{2}\right)}} \quad (12)$$

Thus, for fixed n and $L_{ij} \in (L_{ijr}, L_{ijr-1})\forall$i and j $L_i \in (L_{i,r}, L_{i,r-1}),\forall$ i lot size shipment can be determined by equation (12) and algorithm is designed to determine the optimal solution.

Algorithm

Nearest Neighborhood Algorithm for Vehicle Routing

Step 1: Set vendor's position at origin

Step 2: Calculate the distance between the vendor and each buyer, then select the buyer with the shortest distance. Next, iteratively choose the nearest unvisited buyer to the last selected location.

Step 3: Calculate the total distance traveled so far .if the total distance exceeds the vehicle's Capacity return to the vendor

Step 4: Repeat 1 to 3 steps until all buyers have been visited o the vehicle's capacity is reached.

Algorithm for Optimal Solution

Step 1: Set m=1

Step 2: For each L_{ijk}, k=0,1,2,….,m_{ij} determine Q_{ij}, for all i=0,1,2,…., M and j=1,2…N then Find $JTEC_j(Q_{ij},L_{ijk})$

Step 3: Set $JTEC_j(Q_{ij}^m, L_{ij}^m)=\min JTEC_j(Q_{ij},L_{ijk})$, k=0, 1 ,2,…., m_{ij} and this will be optimal solution for fixed m

Step 4: Compute $JTEC(Q_{ij}^m, L_{ij}^m)=\sum_{j=1}^{N} JTEC_j(Q_{ij},L_{ijk})$ is the expected cost for fixed value of m

Step 5: put m=m+1 and repeat step 2 to 4

Step 6: If $JTEC(Q_{ij}^m, L_{ij}^m, m) \leq JTEC(Q_{ij}^{m-1}, L_{ij}^{m-1}, m-1)$ then go to step 5, otherwise go to

Step 7: Set$(Q_{ij}^*, L_{ij}^*, m)=(Q_{ij}^{m-1}, L_{ij}^{m-1}, m-1)$ and (Q_{ij}^*, L_{ij}^*, m) is the optimal solution

RESULTS AND DISCUSSION

Numerical Analysis

To evaluate the effectiveness of the proposed model, a numerical example is presented involving a supply chain with one vendor and five buyers, each dealing with two items (i.e.,M = 2,N = 5). Table 1 provides the parameters related to the vendor for the respective items. Buyer-specific parameters—including setup cost, holding cost, unit purchase cost, basic consumer demand, unit retail price, and lead time—are detailed in Tables 2 through 8. Transportation-related data includes a fixed cost f=50 units, a per-unit transportation cost C_1 =1.4 and a vehicle distance capacity of 15 km.

Table 9 lists the coordinates or distances from the buyers. The average fuel consumption β is 2 liters per 9 km, and the greenhouse gas (GHG) emission factor γ is 0.003 tons per liter of fuel. The emission tax rate e_1 is set at 2 units. GHG emissions from production are modeled using the following parameters: $a=3\times10^{-7}$ ton-year^2/unit^3,b=0.0012 ton –year/unit^2, c=1.4 ton/unit. MATLAB R2015b has been used as computing platform.

Table 1: Common item data for all items

Item i	A_{vi}	h_{vi}	P_{vi}	C_{vi}(per unit)
1	150	0.1	5000	15
2	120	0.1	5000	20

Table 2: Setup cost of ith product for jth buyer (A_{bij})

Buyer j	Item i=1	Item i=2
1	30	35
2	30	35
3	30	35
4	30	35
5	30	35

Table 3: Holding cost of ith product for jth buyer

Buyer j	Item i=1	Item i=2
1	0.1	0.12
2	0.1	0.12
3	0.1	0.12
4	0.1	0.12
5	0.1	0.12

Table 4: Unit purchase cost of ith product for jth buyer

Buyer j	item i=1	item i=2
1	10	15
2	10	15

3	10	15
4	10	15
5	10	15

Table 5: Basic consumer demand of i^{th} product for j^{th} buyer (α_{ij})

Buyer j	Item i=1	item i=2
1	850	900
2	900	850
3	850	900
4	900	850
5	900	850

Table 6: Positive integer for price sensitive demand of i^{th} product for j^{th} Buyer (β_{ij})

Buyer j	Item i=1	item i=2
1	1	1
2	1	1
3	1	1
4	1	1
5	1	1

Table 7: Unit retail price of i^{th} product for j^{th} buyer (p_{ij})

Buyer j	Item i=1	item i=2
1	250	200
2	200	250
3	250	200
4	200	250
5	250	200

Table 8: Lead time data

Buyer j	Lead time (week)	$C(L_{ij})$
1	8	0
	6	5.6
	4	22.4
2	7	0
	5	7
	4	16.1
3	9	0
	7	5.6
	4	40.6
4	8	0
	6	5.6
	4	22.4
5	7	0
	5	7
	4	16.1

Table 9: Distance data of buyers from vendor

Buyer	Coordinates	Distance from vendor
B1	(3,2)	3.6

B2	(5,1)	5.6
B3	(4,3)	5.09
B4	(8,4)	9.21
B5	(4,4)	5.09

By applying the vehicle routing algorithm to the given data, we observe that two optimal routes are formed: Route 1 — V → B1 → B5 → B3 → V, and Route 2 — V → B2 → B4 → V. Thus, the total number of routes is k=2.

For the initial shipment, m=1 with no crashing cost applied

$C(L_{ij})$, the optimal order quantities are (Q_{11}, Q_{21}) =(538.39,418.36), and the total cost incurred for Buyer 1's items is 6029.56. When the crashing cost increases to 5.6, the optimal quantities become (543.65, 422.84), raising Buyer 1's total cost to 6044.98. Further increasing the crashing cost to 22.4 results in a total cost of 6092.01. Among these, the minimum cost for Buyer 1 is 6029.56; hence, the Joint Expected Total Cost (JTEC) for Buyer 1 is recorded as 6092.56. Following the same approach, the JTEC values for the remaining buyers are $JTEC_2$ = 6024.53, $JTEC_3$ = 6185.16, $JTEC_4$ = 6033.22 and $JTEC_5$ = 6037.03 03 and the total cost of the system be JTEC = 30309.5. This JTEC = 30309.5 is the total cost for the first shipment. For the shipments m = 2 and 3 we get the total costs 31899.48 and 32939.84 respectively.+-

Table 10: Optimal solution for m=1

m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
1	8	(538.39,418.36)	JETC1=6029.56	30309.5
	7	(577.09,393.20)	JETC2=6024.53	
	9	(540.36,420.03)	JETC3=6185.16	
	8	(582.13,396.96)	JETC4=6033.22	
	7	(558.88,392.67)	JETC5=6037.03	
	6	(543.65,422.84)	JETC1=6044.98	
	5	(584.06,398.40)	JETC2=6043.7	
	7	(545.60,424.49)	JETC3=6200.71	
	6	(587.67,401.09)	JETC4=6048.42	
	5	(565.65,397.88)	JETC5=6056.76	
	4	(559.15,436.00)	JETC1=6092.01	
	4	(593.01,405.07)	JETC2=6068.63	
	5	(577.30,451.39)	JETC3=6297.9	
	4	(603.98,413.23)	JETC4=6094.01	
	4	(574.34,404.55)	JETC5=6082.4	

Table 11: Optimal solution for m=2

m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
2	8	(317.52,271.47)	JETC1=6391.79	31899.48
	7	(345.23,253.09)	JETC2=6385.47	
	9	(319.09,272.88)	JETC3=6383.04	
	8	(349.31,256.22)	JETC4=6371.21	
	7	(332.12,262.90)	JETC5=6367.97	
	6	(321.72,275.24)	JETC1=6374.49	
	5	(350.87, 257.42)	JETC2=6416.26	
	7	(323.27,276.64)	JETC3=6407.94	
	6	(353.77,259.65)	JETC4=6393.95	
	5	(337.55,267.47)	JETC5=6398.97	
	4	(334.022,286.27)	JETC1=6449.55	
	4	(358.05,262.94)	JETC2=6456.29	
	5	(348.29,299.04)	JETC3=6563.53	
	4	(366.83,269.67)	JETC4=6462.15	
	4	(344.49,273.23)	JETC5=6439.28	

Table 12: Optimal solution for m=3

m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
3	8	(241.11,213.71)	JETC1=6584.93	32939.84
	7	(263.44,198.67)	JETC2=6586.74	
	9	(242.47,214.93)	JETC3=6596.76	
	8	(266.97,201.41)	JETC4=6617.83	
	7	(252.87,206.75)	JETC5=6553.58	
	6	(244.73,217.00)	JETC1=6617.22	
	5	(268.30,202.43)	JETC2=6626.62	
	7	(246.07,218.22)	JETC3=6628.85	
	6	(270.82,204.39)	JETC4=6649.2	
	5	(257.55,210.68)	JETC5=6593.58	
	4	(255.28,226.61)	JETC1=6714.04	
	4	(274.49,207.23)	JETC2=6675.14	
	5	(267.48,237.69)	JETC3=6829.45	
	4	(282.06,213.07)	JETC4=6743.29	
	4	(263.52,215.69)	JETC5=6644.58	

Sensitivity Analysis

Sensitivity Analysis on Holding Cost

To evaluate the effect of varying holding costs on the optimal solutions of the proposed model, the ordering cost is adjusted by 0%, +10%, +20%, and +30%, with all other parameters held constant. The corresponding changes in the joint total expected cost (JTEC) are summarized in Table 7 and graphically depicted in Figure 3. The results clearly indicate that an increase in the buyer's holding cost leads to a proportional rise in the JTEC, highlighting the sensitivity of the system to holding cost variations.

Table 13: Optimal solution for different holding costs

h_{bij}	m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
0%	1	6	(577.09,393.20)	JETC1=6024.53	30448.01
		5	(558.88,392.67)	JETC2=6037.03	
		7	(545.60,424.49)	JETC3=6200.71	
		6	(559.15,436.00)	JETC4=6092.01	
		5	(603.98,413.23)	JETC5=6094.01	
+10%	1	6	(521.99,420.02)	JETC1=6075.6	30564.72
		5	(561.33,396.66)	JETC2=6075.88	
		7	(523.86,422.67)	JETC3=6239.4	
		6	(564.80,399.33)	JETC4=6122.93	
		5	(543.37,396.13)	JETC5=6119.99	
+20%	1	6	(502.72,390.40)	JETC1=6170.41	31044.53
		5	(541.06,367.31)	JETC2=6168.57	
		7	(504.52,391.93)	JETC3=6343.73	
		6	(544.40,369.79)	JETC4=6179.34	
		5	(523.54,366.82)	JETC5=6182.48	
+30%	1	6	(485.44,376.75)	JETC1=6229.62	31367.55
		5	(522.83,360.26)	JETC2=6227.75	
		7	(487.18,378.23)	JETC3=6410.71	
		6	(526.06,356.66)	JETC4=6238.29	
		5	(505.72,360.80)	JETC5=6216.18	

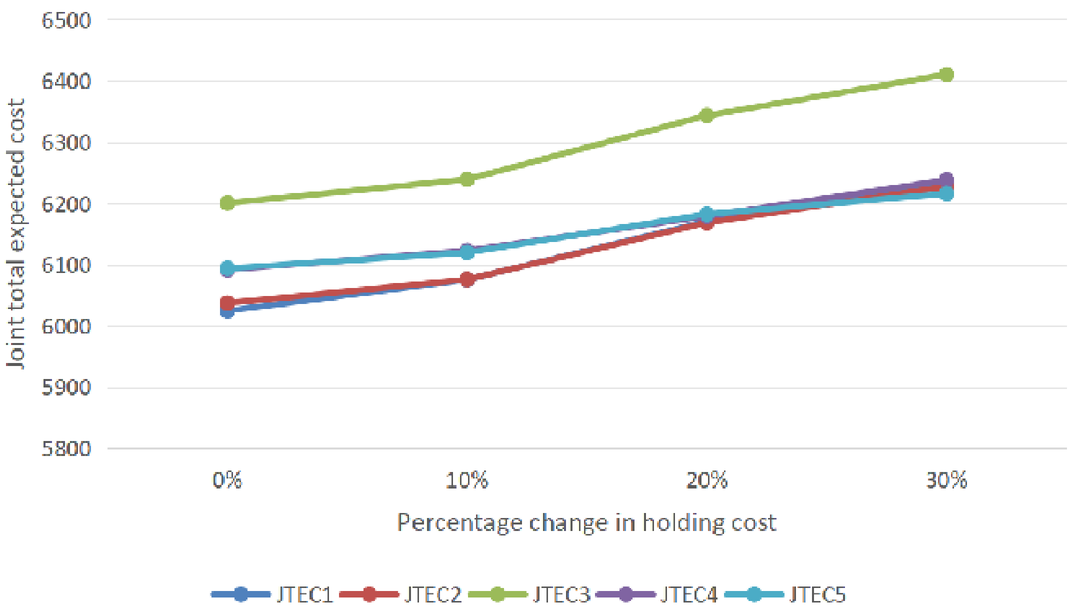

Figure 3: Variation of joint total expected cost with holding cost

Sensitivity Analysis on Setup Cost

To analyze the impact of different setup costs on the optimal solutions of the proposed model, we adjust the setup cost by 0%, +10%, +20%, and +30%, while keeping all other parameters constant. The resulting changes in total cost corresponding to these variations in setup cost are presented in Table 7 and illustrated in Figure 3. The results clearly indicate that an increase in the buyer's setup cost leads to a proportional rise in the JTEC, highlighting the sensitivity of the system to setup cost variations.

Table 14: Optimal solution for different setup costs

A_{vi}	m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
0%	1	6	(577.09,393.20)	JETC1=6024.53	30448.01
		5	(558.88,392.67)	JETC2=6037.03	
		7	(545.60,424.49)	JETC3=6200.71	
		6	(559.15,436.00)	JETC4=6092.01	
		5	(603.98,413.23)	JETC5=6094.01	
+10%	1	6	(557.51,467.29)	JETC1=6083.81	30669.07
		5	(598.74,440.87)	JETC2=6081.83	
		7	(559.41,469.05)	JETC3=6244.83	
		6	(602.26,443.72)	JETC4=6108.54	
		5	(579.90,440.31)	JETC5=6150.06	
+20%	1	6	(571.02,477.29)	JETC1=6119.04	30829.07
		5	(613.06,450.16)	JETC2=6116.71	
		7	(572.88,479.006)	JETC3=6284.24	
		6	(616.50,452.95)	JETC4=6126.21	
		5	(593.80,449.61)	JETC5=6182.87	
+30%	1	6	(584.23,487.07)	JETC1=6156.42	31007.14
		5	(627.05,459.26)	JETC2=6150.87	
		7	(586.04,488.75)	JETC3=6322.78	
		6	(630.42,462.00)	JETC4=6160.14	
		5	(607.39,458.72)	JETC5=6216.93	

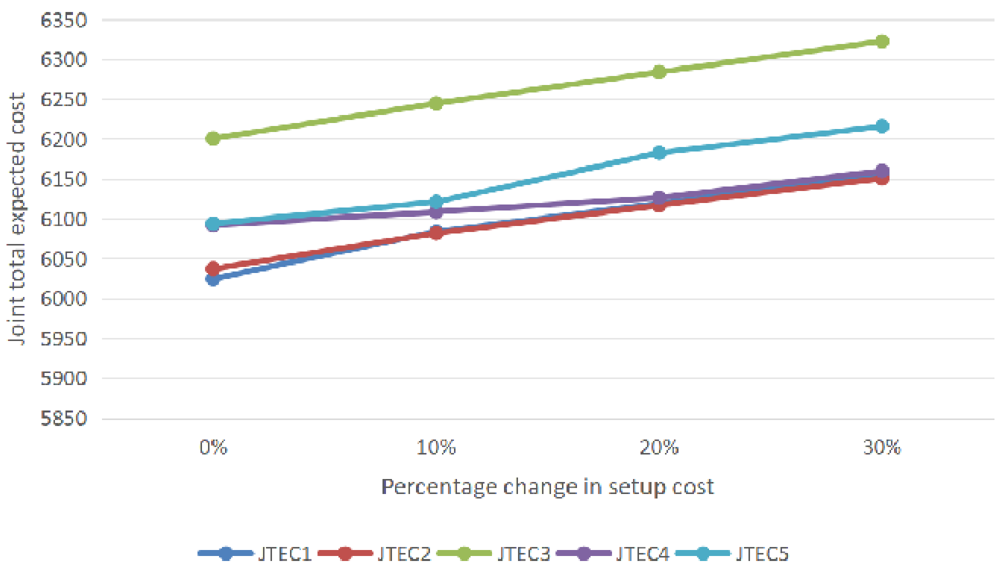

Figure 4: Variation of joint total expected cost with setup cost

Sensitivity Analysis on Ordering Cost

To analyze the impact of different ordering costs on the optimal solutions of the proposed model, we adjust the ordering cost by 0%, +10%, +20%, and +30%, while keeping all other parameters constant. The resulting changes in total cost corresponding to these variations in ordering cost are presented in Table 7 and illustrated in Figure 3. The results clearly indicate that an increase in the buyer's ordering cost leads to a proportional rise in the JTEC, highlighting the sensitivity of the system to ordering cost variations.

Table 15: Optimal solution for different ordering costs

Ab_{ij}	m	L_{ij}	Q_{ij}	$JTEC_i$	JTEC
0%	1	6	(577.09,393.20)	JETC1=6024.53	30448.01
		5	(558.88,392.67)	JETC2=6037.03	
		7	(545.60,424.49)	JETC3=6200.71	
		6	(559.15,436.00)	JETC4=6092.01	
		5	(603.98,413.23)	JETC5=6094.01	
+10%	1	6	(546.45,460.09)	JETC1=6056.85	30456.28
		5	(587.03,434.17)	JETC2=6055	
		7	(548.39,461.86)	JETC3=6214.16	
		6	(590.62,437.07)	JETC4=6109.88	
		5	(568.53,433.61)	JETC5=6110.39	
+20%	1	6	(549.24,463.07)	JETC1=6065.89	30500.92
		5	(589.98,436.94)	JETC2=6063.54	
		7	(551.17,464.84)	JETC3=6223.63	
		6	(593.55,439.82)	JETC4=6118.62	
		5	(571.39,436.38)	JETC5=6119.24	
+30%	1	6	(552.01,466.03)	JETC1=6074.88	30546.15
		5	(592.91,439.70)	JETC2=6072.56	
		7	(553.93,467.79)	JETC3=6233.36	
		6	(596.47,442.55)	JETC4=6128.3	
		5	(574.24,439.14)	JETC5=6130.05	

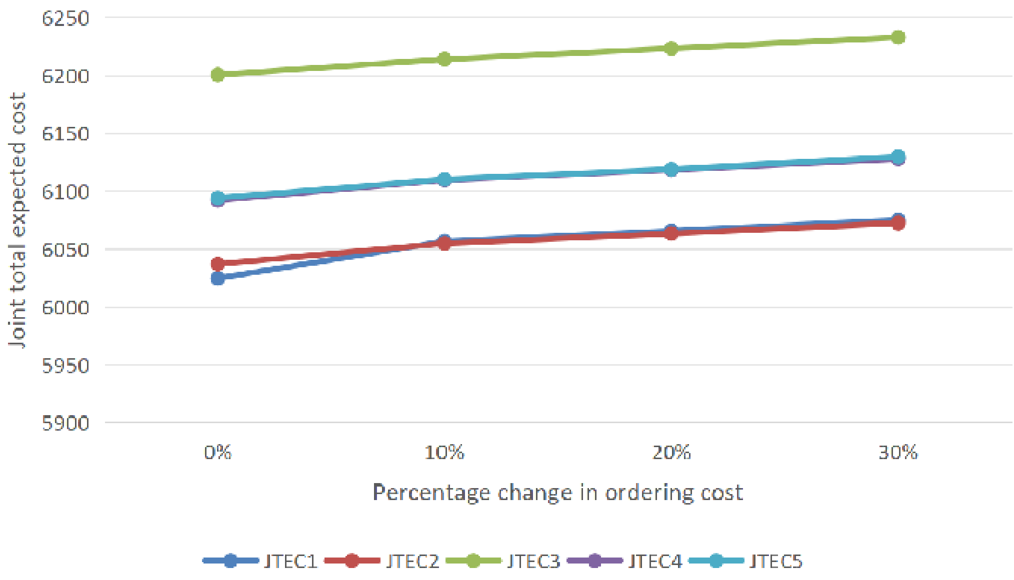

Figure 5: Variation of joint total expected cost with ordering cost

CONCLUSION

This research paper presents an integrated inventory supply chain model involving a single vendor and multiple buyers handling multiple items. The model incorporates key real-world factors, including price-sensitive demand, controllable lead times, transportation costs, and greenhouse gas emission costs associated with both production and transportation. Transportation from the vendor to the buyers is carried out using identical vehicles, following a structured set of delivery routes that define the number of routes and the buyers served in each.

The classical differential calculus technique served to determine the optimal values. The solution procedure addressed the convexity and concavity of the decision variables in detail. The second derivative of the joint total expected cost (JTEC) with respect to Q_{ij} , m, and L_{ij} are shown to be positive which indicates the convexity of the cost function and prove the minimization of the JTEC.

The developed optimization algorithm efficiently determined the optimal order quantities, lead times, and shipment frequencies from the vendor to each buyer. Numerical results reveal that increase in the number of shipments tends to increase the overall cost. Sensitivity analysis shows variations in the joint total expected cost (JTEC) are significantly influenced by holding costs, ordering costs, and setup costs. Specifically, minimizing the JTEC requires buyers to maintain low holding and ordering costs, while the vendor should strive for lower setup costs.

Future Research

However, the model can be extended to more realistic scenarios by incorporating alternative demand patterns and uneven batch sizes. In addition to accounting for greenhouse gas (GHG) emissions from production and transportation, the model can be further enriched by integrating other emission-contributing factors, such as warehousing, packaging, and energy usage. Moreover, penalty costs may be introduced when total emissions exceed predefined regulatory thresholds.

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