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Dynamic Control of Piezoelectric Sandwich Beams with Multi-Frequency Excitations

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ABSTRACT

In this work, we present a non-linear analysis of the dynamics of a sandwich beam with piezoelectric patches under multi-frequency excitations. The model takes into account geometrical non-linearities, piezoelectric non-linearities, and inertia. Hamilton's principle is used to determine the electromechanical equation of motion, and Garlekin's technique is used to determine the equivalent model of the piezoelectric generator. A mathematical methodology based on the multi-scale method for vibration control and stability is reviewed in this work to determine a system of amplitude and phase equations. The control of primary and secondary mode resonances is developed, and feedback effects are analyzed for small and large amplitudes of vibrations of sandwich beams. Frequency response curves are presented and discussed for various gain parameters.

INTRODUCTION

The reduction in the consumption of electronic components has enabled the development of wireless mobile applications. Batteries, which initially encouraged the development of portable electronic devices, have paradoxically become a brake on this progress, particularly because of the associated maintenance problems (recharging and replacement). Since the 90s, research into the control of vibrating structures has led to developing of electronic devices with low rendering requirements. Supplying these small devices with heavy, bulky batteries is not a feasible solution because they have a negative environmental impact and often require very periodic interchangeability. Based on the theory of Euler-Bernoulli beams (belouettar *et al.*, 2006), linear and non-linear mathematical models of the system under consideration are developed to predict the behavior between amplitude and phase under second-order multi-frequency excitation. To contribute to the control of non-linear vibrations, (Daqaq *et al.*, 2009) proposed a non-linear model and demonstrated that the resonance phenomenon can improve the gain harvesting system in terms of vibratory energy. Subsequently, (Triplett *et al.*, 2009) demonstrated that the electromechanical coefficient's non-effect increases the system's output power. In more advanced research, Erturk *et al.* (2011) carried out a series of experimental studies on beams on a macroscopic scale studied the influence of certain parameters such as the linear damping coefficient and the excitation amplitude on the deflection and output voltage. All these previous works show that the degree of non-linearity is always of order 3 or of the duffing type, the order of resolution of the equations according to the disturbance parameter is 1, the position of the piezoelectric patches on the elastic structure and the electrical constants are studied in the linear domain, and finally the geometric shape of the structure is not taken into account.

In this work, we propose a co-rotational formulation of an adaptive sandwich beam. The structure is composed of a viscoelastic core constrained by elastic skins fitted with piezoelectric pads. The special features of the formulation lie in the absence of electrical degrees of freedom to take account of the coupling and in the use of a fractional derivative model for viscoelastic damping. The simplest and most studied geometry for the extraction of vibratory by piezoelectric elements is that of a cantilever beam with one (unimorph) or two (bimorph) piezoelectric layers. Recently, many researchers have devoted their time and efforts to investigating the mechanical and vibrational behavior of both micro- and nano-structures (Younis & Nayfeh, 2003; Farokhi *et al.*, 2013; Askari, 2014; Askari *et al.,* 2014a; Farokhi & Ghayesh, 2015a; Ghayesh, 2015b; Ghayesh, (2015c); Ghayesh & Farokhi, 2015; Han *et al.,* 2015; Ghorbanpour Arani *et al.,* 2016). A few more recent research works in this area include the study of and functionally graded microbeams carrying microparticles in a thermal environment by Shenas et al. 2016 and the size-dependent behavior of functionally graded microbeams using various shear deformation theories by Trinh *et al.* (2016), the size dependent vibration of a nickel cantilever microbeam by Lei *et al.* (2016), the free flexural vibration of geometrically imperfect functionally graded beams by Dehrouyeh-Semnani *et al.* (2016), and the size-dependent behavior of functionally graded microbeams using various shear deformation theories by Shafiei *et al.* (2016).

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LITERATURE REVIEW

Most of the social science research articles include a literature review. It usually contains the significant contribution of past papers to justify the adopted theory and variables. Further to develop hypotheses. It can be divided into subsections. Literature should be written concisely in detail by maintaining continuity of the texts and cited the original work following APA in-text citation format, e.g., Single authored document (Author's Surname, 2021); double authored document (1st Author's Surname & 2nd Author's Surname, 2021); Multiple authored document (1st Author's Surname, *et al.*, 2021).

MATERIALS AND METHODS

Mathematical Modelling

In this section, the geometry of the problem considered is that of a beam in a bimorph configuration: the upper and lower layers with patches. Modeling a piezo/elastic/piezo sandwich beam in the vibration regime gives rise to a system of partial differential equations with a high degree of non-linearity (Horel, 2014).

General Displacement Field

$$\begin{cases} u^i(x,y,z,t) = u^i(x,y,t) + (z - z_i)\varphi_x \\ v^i(x,y,z,t) = v^i(x,y,t) + (z - z_i)\varphi_y \\ w^i(x,y,z,t) = w^i(x,y,t) \end{cases} \tag{1}$$

With $i = e, s, A$: s upper piezoelectric patch layer (sensor) e = elastic intermediate layer (heart) A = lower piezoelectric patch layer (actuator),

φ_x, φ_y = rotations of the normal of the median plane of the central layer

z_i = ordinate of the median axis of the i[th] layer

$$\begin{cases} \varphi_x(x,y) = -\dfrac{\partial w}{\partial x} \\ \varphi_y(x,y) = -\dfrac{\partial w}{\partial y} \\ \varphi_z(x,y) = -\dfrac{\partial w}{\partial z} = 0 \end{cases} \tag{2}$$

and $$z_S = -z_A = \frac{1}{2}(h_e + h_s) \tag{3}$$

The kinematic connection conditions at the sandwich interfaces have the form

$$\begin{cases} u_S(x,y,\dfrac{h_s}{2}) = u_e(x,y,\dfrac{h_s}{2}) \\ u_A(x,y,-\dfrac{h_s}{2}) = u_e(x,y,-\dfrac{h_s}{2}) \end{cases} \tag{4}$$

Hence the displacment fields in the layers

$$\begin{cases} U^i(x,y,z,t) = u_0^i(x,y,t) - (z - \dfrac{h_A + h_S}{2})\omega_{,x} \\ U^e(x,y,z,t) = u_0(x,y,t) + z\varphi(x,y,t) \\ w^i(x,y,z,t) = w_0(x,y,t) \end{cases} \tag{5}$$

With $$u_0^i(x,y,t) \pm u_0 + \frac{h_e}{2}\varphi_x - \frac{h_S}{2}\omega_{,x} \quad i = s, A \tag{6}$$

Deformation Fields

The deformation fields are derived from displacement fields. Considering uniaxial displacements, we have:

$$\begin{cases} \varepsilon_{1S}(x,y,t) = u_{0,x} + \dfrac{h_e}{2}\varphi_{,x} - \dfrac{h_S}{2}w_{,xx} - (z - z_S)w_{,xx} + \dfrac{1}{2}w^2{}_{,x} \\ \varepsilon_{1e}(x,y,t) = u_{0,x} + z\varphi_{,x} \\ \varepsilon_{1A}(x,y,t) = u_{0,x} - \dfrac{h_e}{2}\varphi_{,x} + \dfrac{h_S}{2}w_{,xx} - (z - z_A)w_{,xx} + \dfrac{1}{2}w^2{}_{,x} \end{cases} \tag{7}$$

Plane Stress States

At any point M under plane stress, the elasticity equation can be written as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ 0 \\ 0 \\ 0 \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11}^E & C_{12}^E & C_{13}^E & 0 & 0 & C_{16}^E \\ C_{12}^E & C_{22}^E & C_{23}^E & 0 & 0 & C_{26}^E \\ C_{13}^E & C_{23}^E & C_{33}^E & 0 & 0 & C_{36}^E \\ 0 & 0 & 0 & C_{44}^E & C_{45}^E & 0 \\ 0 & 0 & 0 & C_{45}^E & C_{55}^E & 0 \\ C_{16}^E & C_{26}^E & C_{36}^E & 0 & 0 & C_{66}^E \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ 0 \\ 0 \\ \varepsilon_6 \end{Bmatrix} \tag{8}$$

$$\begin{cases} \sigma_1 = \left(C_{11}^E - \dfrac{(C_{13}^E)^2}{C_{33}^E} \right)\varepsilon_1 + \left(C_{12}^E - \dfrac{C_{13}^E C_{23}^E}{C_{33}^E} \right)\varepsilon_2 + \left(C_{16}^E - \dfrac{C_{13}^E C_{36}^E}{C_{33}^E} \right)\varepsilon_6 \\ \sigma_2 = \left(C_{12}^E - \dfrac{C_{23}^E C_{13}^E}{C_{33}^E} \right)\varepsilon_1 + \left(C_{22}^E - \dfrac{(C_{23}^E)^2}{C_{33}^E} \right)\varepsilon_2 + \left(C_{26}^E - \dfrac{C_{23}^E C_{36}^E}{C_{33}^E} \right)\varepsilon_6 \\ \sigma_6 = \left(C_{16}^E - \dfrac{C_{36}^E C_{13}^E}{C_{33}^E} \right)\varepsilon_1 + \left(C_{26}^E - \dfrac{C_{36}^E C_{23}^E}{C_{33}^E} \right)\varepsilon_2 + \left(C_{66}^E - \dfrac{(C_{36}^E)^2}{C_{33}^E} \right)\varepsilon_6 \end{cases} \tag{9}$$

Let's put $C_{11}^* = \left(C_{11}^E - \dfrac{(C_{13}^E)^2}{C_{33}^E} \right)$, $C_{22}^* = \left(C_{22}^E - \dfrac{(C_{23}^E)^2}{C_{33}^E} \right)$, $C_{66}^* = \left(C_{66}^E - \dfrac{(C_{36}^E)^2}{C_{33}^E} \right)$,

$$C_{12}^* = C_{12}^E - \frac{C_{13}^E C_{23}^E}{C_{33}^E}, \quad C_{16}^* = C_{16}^E - \frac{C_{36}^E C_{13}^E}{C_{33}^E}, \quad C_{26}^* = C_{26}^E - \frac{C_{23}^E C_{36}^E}{C_{33}^E} \tag{10}$$

Electric Potential and Electric Field in the Viscoelastic Core

The electrical potential in the elastic layer Φ is a function of the electrical potentials at the surface (Opinathan *et al.*, 2000). Note Φ^S et Φ^A the electrical potentials on the (Sensor) and (Actuator) surfaces respectively.

Whether in open or closed circuit Φ=0and for a heart with open circuit Φ is unknown

$$\int_0^L D_3(\pm \frac{h_S}{2})\,dx = 0 \tag{11}$$

Variational Formulation

The dynamic equations of a continuum with piezoelectric patches are deduced from Hamilton's principle, where the Lagrangian and virtual work are adapted to include electrical and mechanical contributions.

$$\delta T - \delta H + \delta W = 0 \tag{13}$$

$$\delta T = \frac{1}{2}\int_{v_i} \rho_i(\delta \dot{u}_i + \delta \dot{w}_i)dv = (\rho s)_{eq}\int_0^a \left(\dot{u}_i \ddot{u} + \dot{w}\ddot{w}_i \right)dx \tag{14}$$

$$W = \left[\int_{s_i} (F_x u_i + F_z w_i) \right]_0^a + \int_{v_i} (f_x u_i + f_z w_i)\,dv \tag{15}$$

$$\begin{cases} N_S = (E_e S_e + C_{11}^* S_S)u_{0,x} + C_{11}^* S_S w_{,x}^2 + e_{31}^* \dfrac{S_S}{h_s}(\phi^S - \phi^A) \\ M_\varphi = \dfrac{h_e^2}{2}C_{11}^* S_S \varphi_{,x} + (E_e I_e - \dfrac{h_e}{2}h_s C_{11}^* S_S)w_{,xx} + e_{31}^* S_S \dfrac{h_e}{2h_S}(\phi^S + \phi^A) \\ M_\omega = \dfrac{h_e}{2}h_s C_{11}^* S_S \varphi_{,x} + C_{11}^*(I_S + I_A - S_S\dfrac{h_s^2}{2})w_{,xx} + e_{31}^* S_S \dfrac{1}{2}(\phi^S + \phi^A) \end{cases} \tag{16}$$

Controls Taken into Account

Several analytical methods are used to control a vibrating structure. So when the structure vibrates, the piezoelectric sensor layer generates an output voltage which is amplified and fed back via a direct proportional law, or velocity control law. The control can be expressed by the proportional derivative (PD) law defined by:

$$\phi^A = G_d\phi^S + G_v\dot{\phi}^S \tag{17}$$

Condensation of Electrical Potential

$$\begin{cases} N=\left(E_eS_e+2C_{11}^*S_S+\lambda\right)u_{0,x}+\lambda\frac{h_e}{2}\varphi_{,x}+\frac{1}{2}(2C_{11}^*S_S+\lambda)w_{,x}^2-e_{31}^*\frac{S_S}{h_S}\phi^A \\ M\varphi=\lambda\frac{h_e}{2}u_{0,x}+\frac{h_e^2}{4}(2C_{11}^*S_S+\lambda)\varphi_{,x}+\left(E_eI_e-\frac{h_e}{4}h_S(2C_{11}^*S_S+\lambda)\right)w_{,xx}+\lambda\frac{h_e}{4}w_{,x}^2+e_{31}^*S_S\frac{h_e}{2h_S}\phi^A \\ M_w=\lambda\frac{h_S}{2}u_{0,x}+\frac{h_e^2}{4}(2C_{11}^*S_S+\lambda)\varphi_{,x}+\left(C_{11}^*I_S+C_{11}^*I_A-\frac{h_S^2}{4}(2C_{11}^*S_S+\lambda)\right)w_{,xx}+\lambda\frac{h_S}{4}w_{,x}^2+e_{31}^*S_S\frac{\phi^A}{2} \end{cases} \tag{18}$$

Axial displacement due to static condensation . Applying Von Karman's dynamic model gives

$$(\rho S)_{eq}\ddot{w}-M_{w,xx}N(t)M_{,xx}=F \tag{19}$$

So the equation of motion of our beam is written as:

$$(\rho S)_{eq}\ddot{w}+(EI)_{eq}w_{,xxxx}-N(t)w_{,xx}+\lambda\frac{h_S}{2}(w_{,xx}^2+w_{,x}w_{,xxx})+$$
$$\lambda\frac{h_S}{2}G_v\left(\dot{w}_{,x}w_{,xxx}+w_{,x}\dot{w}_{,xxx}+2\dot{w}_{,xx}w_{,xx}-\frac{h_S}{2}\dot{w}_{,xxx}\right)=F \tag{20}$$

The transverse displacement w according to the Galerkin's approximation is written by:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\tau_{mn}(t)\psi_{mn}(x,y)$$

Where $w_k(x, y)$: free vibration modes with boundary conditions taken into account

$\tau_k(t)$: Time amplitudes. In order to study the behaviour of the piezo /elastic /piezo patch effect, we will equate the fundamental vibration by:

$$\ddot{\tau}+2\mu\dot{\tau}+w_l^2\tau+\alpha_2\tau^2+\alpha_3\tau^3+\alpha_4\tau\dot{\tau}+\alpha_5\tau^2\dot{\tau}=F(t) \tag{21}$$

With

$$\begin{cases} \alpha_2=\lambda\frac{h_S}{2M}(1+G_d)\int_0^a\left(\psi_{,xxx}\psi_{,x}+\psi_{,xx}^2\right)\psi(x,y)dx+\frac{\lambda h_S}{2LM}G_d\int_0^a\psi_{,xx}dx\int_0^a\psi_{,xx}\psi(x,y)dx \\ \alpha_3=\frac{1}{aM}\left(2C_{11}^*S_S+\lambda(1-G_d)\right)\int_0^L w_{,x}^2dx\int_0^a\psi_{,xx}\psi(x,y)dx \\ \alpha_4=\lambda\frac{h_S}{2M}G_v\left[\frac{1}{2a}\int_0^a\psi_{,xx}\psi_{,x}dx+\int_0^a\left(\psi_{,xxx}\psi_{,x}+\psi_{,xx}^2\right)\psi(x,y)dx\right] \\ \alpha_5=\frac{\lambda}{2M}G_v\int_0^a\left(\psi_{,xxx}\psi_{,x}+\psi_{,xx}^2\right)dx\int_0^a\psi_{,xx}\psi(x,y)dx \end{cases}$$

$$\begin{cases} 2\mu=\frac{\lambda h_S^2}{2M}G_v\int_0^a\psi_{,xxxx}\psi(x,y)dx \\ w_L^2=\frac{\left(C_{11}^*I_S+C_{11}^*I_A-\frac{h_S^2}{4}\left(2C_{11}^*S_S+\lambda(1+G_d)\right)\right)}{M}\int_0^a\psi_{,xxxx}\psi(x,y)dx \\ M=(\rho S)_{eq}\int_0^a\psi^2(x,y)dx \end{cases} \tag{22}$$

$$F=\frac{1}{M}\int_0^a f(x)\psi(x,y)dx$$

Multiple Scale Method

To solve equation 21, we use the multi-scale method which consists of developing the electromechanical quantities in whole series according to the increasing powers of the disturbance parameter (8).

We define de time scale as :

$$\begin{cases} T_n=\varepsilon^n t=\varepsilon^0 t+\varepsilon^1 t+.....=T_0+T_1+T_2 \\ T_0=\varepsilon^0 t \end{cases}$$
$$\tau(t,\varepsilon)=\varepsilon^0\tau_0(T_0,T_1+.....)+\varepsilon^1\tau_1(T_0,T_1+.....)+............... \tag{23}$$
$$\begin{cases} \frac{d}{d_n}=\varepsilon^0\frac{\partial}{\partial T_0}+\varepsilon^1\frac{\partial}{\partial T_1}+\varepsilon^2\frac{\partial}{\partial T_2}+.......... \\ \frac{d^2}{dT_n^2}=\varepsilon^0 D_0^2+2\varepsilon^1 D_0D_1+.......... \end{cases}$$

Primary Resonance

Thus, the equivalent electromechanical laws defined become, with free vibration modes $\psi_k(x,y)$

$$\begin{cases} N_S=(E_eS_e+C_{11}^*S_S)u_{0,x}+C_{11}^*S_Sw_{,x}^2+e_{31}^*\frac{S_S}{h_s}(\phi^S-\phi^A) \\ M_\varphi=\frac{h_e^2}{2}C_{11}^*S_S\varphi_{,x}+(E_eI_e-\frac{h_e}{2}h_sC_{11}^*S_S)w_{,xx}+e_{31}^*S_S\frac{h_e}{2h_S}(\phi^S+\phi^A) \\ M_\omega=\frac{h_e}{2}h_sC_{11}^*S_S\varphi_{,x}+C_{11}^*(I_S+I_A-S_S\frac{h_s^2}{2})w_{,xx}+e_{31}^*S_S\frac{1}{2}(\phi^S+\phi^A) \end{cases} \tag{24}$$

Control can be expressed by the derivative proportional law (PD) defined by :

$$\phi^A=G_d\phi^S+G_v\dot{\phi}^S \tag{25}$$

Solving the homogeneous equation gives and by replacing in the second expression we obtain

$$D_0^2\tau_1+\omega_L^2\tau_1=\left[-2i\omega_L\left(A'+\mu A\right)-3\alpha_3A^2\bar{A}-i\omega_L\alpha_5A^2\bar{A}\right]e^{i\omega_LT_0}.$$
$$+\frac{F_1}{2}e^{i\omega_1T_0}+\frac{F_2}{2}e^{i\omega_2T_0}+\frac{F_3}{2}e^{i\omega_3T_0}+cc \tag{26}$$

After eliminating the secular terms, we obtain :

$$\left[2i\omega_L\left(A'+\mu A\right)+3\alpha_3A^2\bar{A}+i\omega_L\alpha_5A^2\bar{A}\right]=$$
$$\frac{F_1}{2}e^{i(\omega_1-i\omega_L)T_0}+\frac{F_2}{2}e^{i(\omega_2-i\omega_L)T_0}+\frac{F_3}{2}e^{i(\omega_3-i\omega_L)T_0} \tag{27}$$

with $\bar{A}(T_1)$ the conjugate of $A(T_1)$, $A'=D_1A$, the polar f of $A(T_1)$ is given by

$$A(T_1)=\frac{1}{2}ae^{i\beta},\ \bar{A}(T_1)=\frac{1}{2}ae^{-i\beta},\ A'=\frac{1}{2}\left(a'+ia\beta'\right)e^{i\beta} \tag{28}$$

(α and β real fonctions). By injecting these expressions into (27) we obtain:

$$\left[i\omega_L\left(a'+i\beta'+\mu a\right)+\frac{3}{8}\alpha_3a^3+i\frac{1}{8}\omega_L\alpha_5a^3\right]e^{i\beta T_1}=\frac{F_1}{2}e^{i(\omega_1-i\omega_L)T_0}+\frac{F_2}{2}e^{i(\omega_2-i\omega_L)T_0}+\frac{F_3}{2}e^{i(\omega_2-i\omega_L)T_0} \tag{28}$$

$$\begin{cases} a'+\mu a+\frac{1}{8}\alpha_5a^3=\frac{F_1}{2\omega_L}\sin\gamma_1+\frac{F_2}{2\omega_L}\sin\gamma_2+\frac{F_3}{2\omega_L}\sin\gamma_3 \\ \gamma_i'-a\left(\sigma-\frac{3}{8\omega_L}\alpha_3a^2\right)=-\frac{F_1}{2\omega_L}\cos\gamma_1-\frac{F_2}{2\omega_L}\cos\gamma_2-\frac{F_3}{2\omega_L}\cos\gamma_3 \end{cases} \tag{29}$$

$$\sigma=\frac{3}{8w_L}\alpha_3a^2\pm\sqrt{\frac{1}{4a^2w_L^2}\left(F_1^2+F_2^2+F_3^2\right)+\frac{1}{2a^2w_L^2}[F_1F_2+F_1F_3+F_2F_3]-\left(\mu+\frac{1}{8}\alpha_5a^2\right)^2} \tag{30}$$

Stability Study

The solutions of the equilibrium points associated with the primary resonance are obtained by solving the system of equations.

$$\begin{cases} \mu+\frac{1}{8}\alpha_5a^2=\frac{F_1}{2aw_L}\sin\gamma_1+\frac{F_2}{2aw_L}\sin\gamma_2 \\ -\sigma-\frac{3}{8w_L}\alpha_3a^2=\frac{F_1}{2aw_L}\cos\gamma_1+\frac{F_2}{2aw_L}\cos\gamma_2 \end{cases} \tag{31}$$

Determine the given values of the Jacobian matrix J

$$J=\begin{bmatrix} -\mu-\frac{3}{8}\alpha_5 a^2 & -a\sigma_i+\frac{3}{8w_L}\alpha_3 a^3 \\ \sigma_i+\frac{9}{8w_L}\alpha_3 a^2 & -\mu-\frac{1}{8}\alpha_5 a^2 \end{bmatrix} \qquad (32)$$

$$\lambda_i=\sqrt{\left(\mu+\frac{1}{4}\alpha_5 a^2\right)\pm\left[4\left(\mu^2+\frac{1}{8}\mu\alpha_5 a^2+\frac{1}{16}\alpha_5{}^2a^4\right)-4\left(\mu^2+\frac{3}{64}\alpha_5{}^2a^4\right)\left(\sigma+\frac{9}{8w_L}\alpha_3 a^2\right)\left(-a\sigma+\frac{3}{8w_L}\alpha_3 a^3\right)\right]}$$

Secondary Resonance

$$\ddot{\tau}+\omega_L^2\tau=-\varepsilon\left[2\mu\dot{\tau}+w_L^2\tau+\alpha_2\tau^2+\alpha_3\tau^3+\alpha_4\tau\dot{\tau}+\alpha_5\tau^2\dot{\tau}\right]+$$

$$F_1\cos(\omega_1 T_0+\gamma_1)+F_2\cos(\omega_2 T_0+\gamma_2)+F_3\cos(\omega_3 T_0+\gamma_3) \qquad (33)$$

$$\begin{cases} \dfrac{d}{d_n}=\varepsilon^0\dfrac{\partial}{\partial T_0}+\varepsilon^1\dfrac{\partial}{\partial T_1}+\varepsilon^2\dfrac{\partial}{\partial T_2}+.......... \\ \dfrac{d^2}{dT_n^2}=\varepsilon^0 D_0^2+2\varepsilon^1 D_0 D_1+.......... \end{cases}$$

$$\left(D_0^2+2\varepsilon D_0^2+....\right)\left(\tau_0+\varepsilon\tau_1+....\right)+w_L^2\left(\tau_0+\varepsilon\tau_1+...\right)+2\hat{\mu}\left(D_0^2+2\varepsilon D_0^2+....\right)$$

$$\left(\tau_0+\varepsilon\tau_1+....\right)+\alpha_2\left(\tau_0+\varepsilon\tau_1+....\right)^2+\alpha_3\left(\tau_0+\varepsilon\tau_1+....\right)^3+$$

$$\alpha_4\left(\tau_0+\varepsilon\tau_1+....\right)\left(D_0+\varepsilon^1 D_1+....\right)\left(\varepsilon^0\tau_0+\varepsilon\tau_1+....\right)\alpha_5\left(\tau_0+\varepsilon\tau_1+....\right)^2\left(D_0+\varepsilon^1 D_1+....\right)\left(\tau_0+\varepsilon\tau_1+....\right)=$$

$$F_1\cos\left(\omega_1\tau+\gamma_1\right)+F_2\cos\left(\omega_2\tau+\gamma_2\right)+F_3\cos\left(\omega_3\tau+\gamma_3\right) \qquad (34)$$

$$\Rightarrow\begin{cases} D_0^2\tau_0+w_L^2\tau_0=F_1\cos\left(\omega_1 t+\gamma_1\right)+F_2\cos\left(\omega_2 t+\gamma_2\right)+F_3\cos\left(\omega_3 t+\gamma_3\right) \\ D_0^2\tau_1+w_L^2\tau_1=-2D_1\left(D_0\tau_0+\hat{\mu}\tau_0{}^{\circ}\right)+\alpha_2\tau_0^2+\alpha_3\tau_0^3+\alpha_4\tau_0 D_0\tau_0+\alpha_5\tau_0^2 D_0\tau_0 \end{cases} \qquad (35)$$

$$\tau_0(t)=A(T_1)e^{i(w_1+w_2+w_3)T_0-\beta-\gamma_i)}+2\sum_{n=0}^{3}K_n e^{iw_n T_0}+cc$$

Numerical Simulation and Discussion

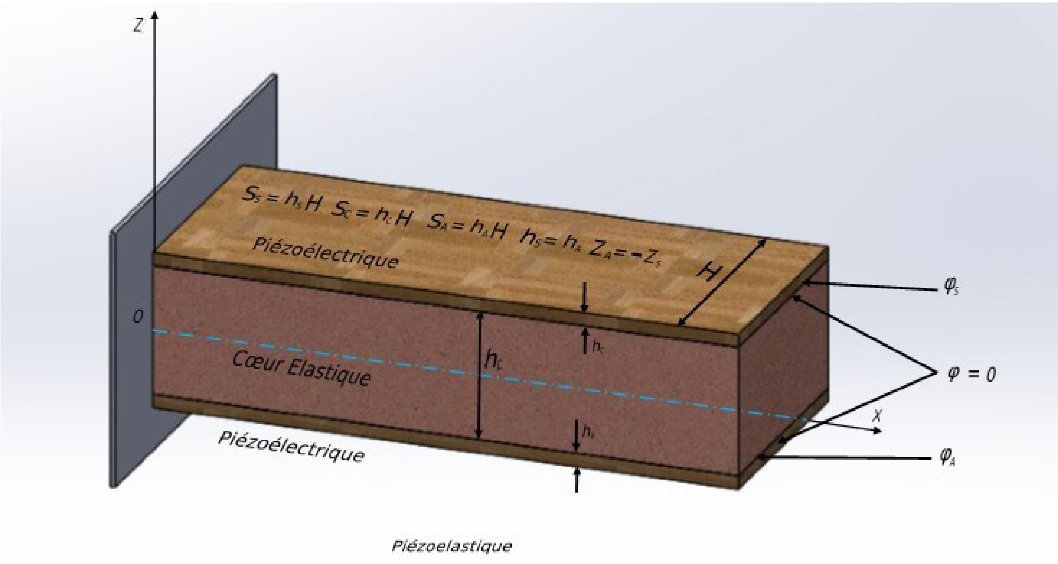

Figure 1: Piezo/Visco/Piézo beams

Table 1: Properties of the FGM skin forming the core and the piezoelectric pads on the surfaces PZT /FGL/ PZT piezo beam

Physical properties	**MGF skin**	**Materials (PZT) Sensor / Actuator**
Length (m)	L_b=0.18	L_c=L_a=0.3
Width (m)	B_b=0.025	B_c=B_a=0.05
Thickness (m)	h_b=0.07	h_c=h_a=0,0008
Density (Kg /m^3)	$\rho_m=9246$ $\rho_c=2730$	E_p=69,5
Young's modulus (Gpa)	E_m=305,9 E_c=422,2	
Strain constant (PZT) (m/V)		$d_{31}=153\times10^{(-12)}$
Stress constant (PZT) (m/V)		$g_{31}=12,7\times10^{(-3)}$

Primary Resonance

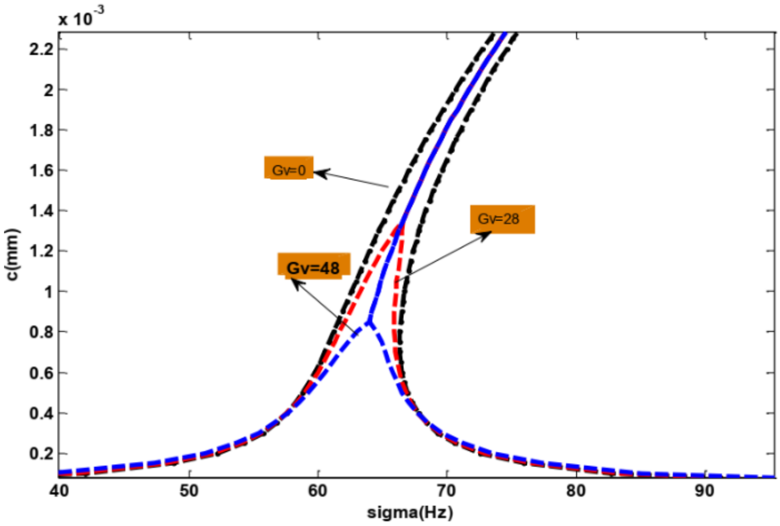

Figure 2: Nonlinear frequency-amplitude for Gd (-0,5 ;-0,2 ;0 ; 0,2, 0,5, 0,8) Gv= 30. γ1-γ2 = 0 , γ1-γ3 = 0 , γ2-γ3 =0

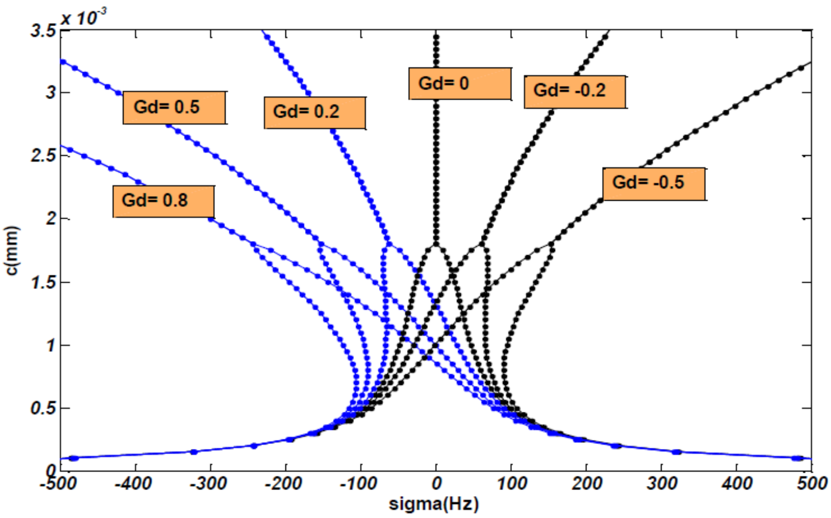

Figure 3: Nonlinear frequency-amplitude for Gv (0 ;28 ;48) Gd= 0.05 γ1-γ2 = 60° , γ1-γ3 = 45° , γ2-γ3 = 30°

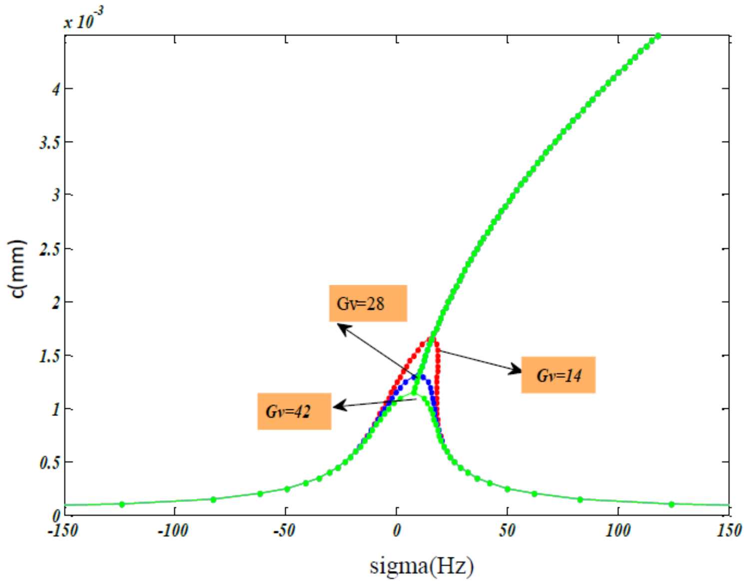

Figure 4: Nonlinear frequency-amplitude for Gv (14 ;28;42) Gd= 0.05. γ1-γ2 = 90° , γ1-γ3 = 90° , γ2-γ3 = 90°

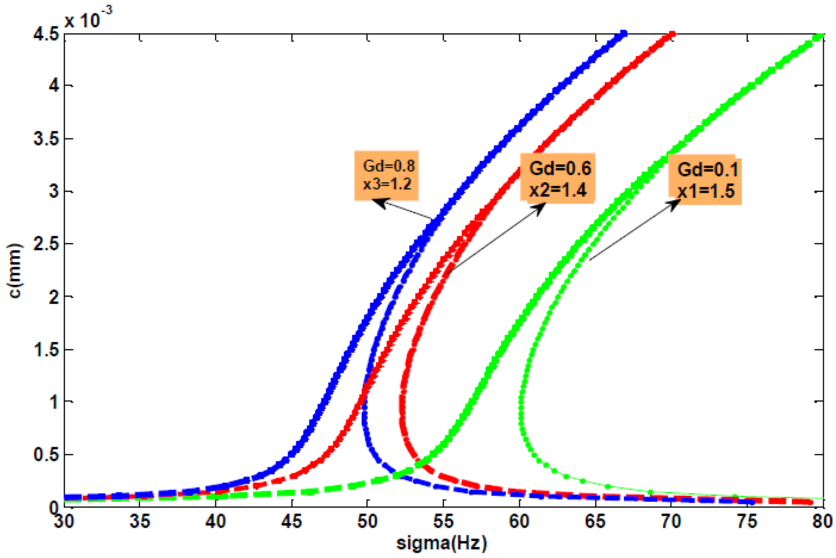

Figure 5: Nonlinear frequency-amplitude for Gd(0,1 ;0,6, 0,8) Gv= 30. X(1.5 ;1.4 ;1.2)

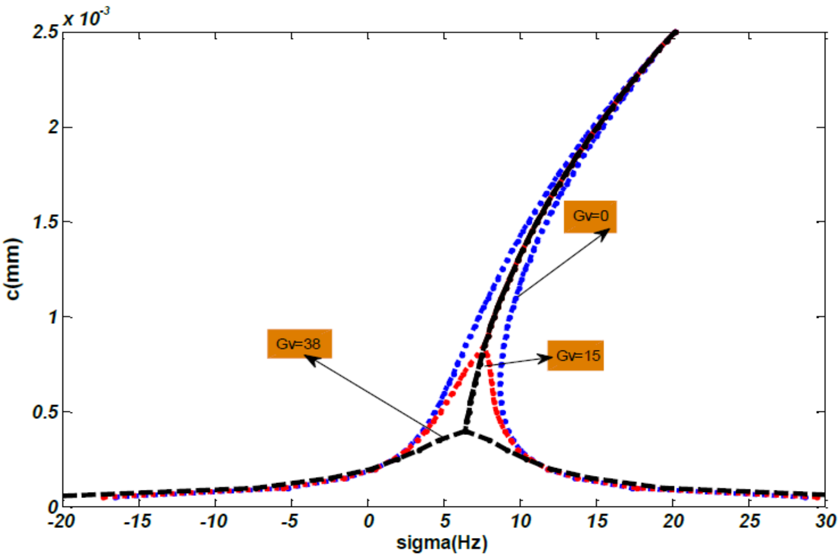

Figure 6: Nonlinear frequency-amplitude for Gv (0 ; 15 ; 38) Gd= 0.005. X (1.1 ; 1.3 ; 1.4)

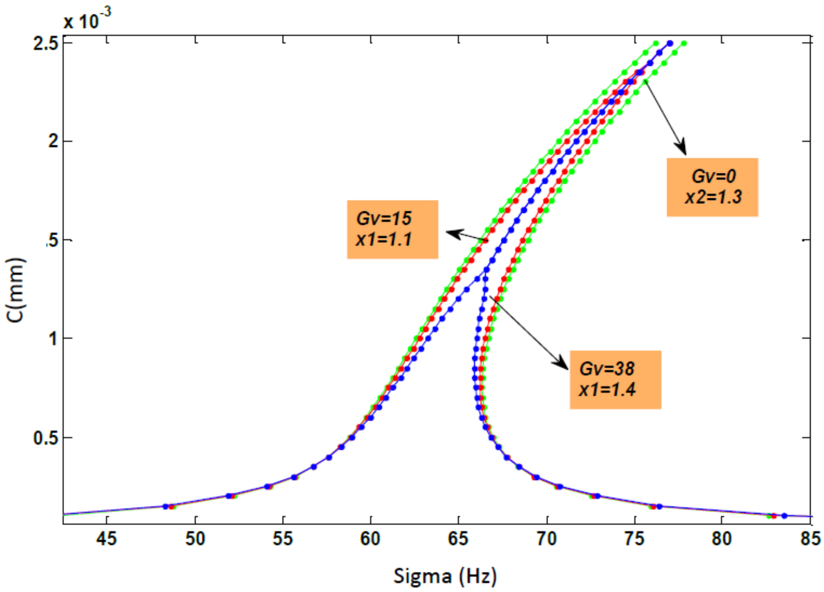

Figure 7: Nonlinear frequency-amplitude for Gv (0 ; 15 ; 38) Gd= 0.005. X (1.1 ; 1.3 ; 1.4)

We can see from these curves that the gain parameters Gd have a considerable influence on the vibration amplitudes. Indeed, for negative values of gain Gd, Gd (-0.5 ; -0.2), the curves obtained are oriented towards high frequencies, which reflects the stiffening nature of the beam. On the other hand, for positive values of Gd (0.2; 0.5; 0.8), the curves are oriented towards low frequencies with relatively large amplitudes, reflecting the beam's softening behaviour. On the other hand, for positive values of Gd (0.2; 0.5; 0.8), the curves are oriented towards low frequencies with relatively large amplitudes, which reflects the softening behaviour of the beam. For the zero value of Gd (0), the curve obtained describes a more linear behaviour, with the amplitude of vibration increasing rapidly, then decreasing just as quickly and stabilising, which would explain why the plate no longer vibrates. We can see that the angles between the different forces applied to the beam play an important role in the behaviour of the structure. Indeed, when the difference in angle is zero, the curves obtained are practically grouped together and, depending on the values of Gd, the beam may stiffen or soften.

Figure 4 shows the frequency-amplitude behaviour of the beam as a function of the parameter Gv and the angles between the different forces. It can be seen that the amplitude decreases with Gv. The curves have the same orientation and are symmetrical about the same axis. Our results can be explained by the fact that the greater the gain parameter Gv, the more the vibration amplitude decreases. On the other hand, for small values of Gv, the amplitude is large. The curve shows the amplitude-frequency curves of the non-linear vibrations of a beam on which a direct proportional control has been carried out with Gd as the control parameter. It can be seen that the amplitude of the vibration decreases as the values of Gd become larger, and also that the amplitude of the vibrations decreases with the points where the forces are applied; in fact, the further these points are from the origin of the reference frame, the greater the amplitude of the vibration.The same amplitudes can be observed whatever the gain parameter Gd. This could lead us to say that the points of application of the forces play a predominant role in the frequency responses. We can also see that the curves are oriented towards high frequencies, depending on the values of Gd , where the beam acquires a stiffening or resisting character. In this case, the beam vibrates more for longer, which makes it less stable. The curves illustrate the non-linear behaviour of the sandwich beam under the influence of the velocity gain Gv, we can see that the curves have the same orientation and are symmetrical about the same axis. The amplitudes evolve in the opposite direction to the parameter Gv, i.e. the amplitudes decrease with the values of Gv.

CONCLUSION

In this work, the nonlinear performances of the piezoelectric parametrically which includes geometric , inertia and electromechanical coupling nonlinearities, is studied. A mathematical model of the active control of the sandwich beam piezoelectric/elastic/piezoelectric triangular plate is developed in this work. Direct proportional control and velocity control are studied through the use of the piezoelectric sensor layer and the piezoelectric actuator, the central layer of which is made of ST37 steel. Thanks to the electromechanical coupling, we have modelled the dynamics of a beam by a non-linear partial differential equation depending on the various control gain parameters. Based on the Galerkin approximation, a second-order non-linear differential equation is obtained with strong non-linearities. The multi-scale is used to solve the non-linear differential equation and the frequency-amplitude relations are deduced in the case of primary and secondary resonances and secondary resonances. The analytical relationships of the Jacobian eigenvalues, which define the stability zone and the instability zone of the plate, are given in the context of a static study.

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