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Analytical Solution for Optimum Design of Nozzle Discharge Lines Based on Hydraulic Considerations

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ABSTRACT

This study focuses on finding the best design of nozzles used in the water discharge pipelines delivering free jets based on hydraulic considerations. An analytical solution is carried out to determine the best design for these nozzles. The solution depends on the derivative method applied to the energy head concerning the nozzle diameter to maximize the power of the jet. The equation derived for computing the best pipe-nozzle diameter ratio D/d in discharge lines applies over practical ranges of relative distance L/D (50-500) and relative roughness e/D (0.01-0.05). An illustrative example is solved and gives D/d = 1.8858 (closely equal to 1.9), the deviation is 0.77%, which is firmly accepted. The equation holds good for D/d from 1.4 to 3.584 and could be applied to compute D/d ratios without the need for a computer program. In the derived equation, D/d is a function of relative distance L/D, and friction factor F with negligible computational errors. The derived equation when compared with the conventional one, shows an increase in the computed power of the jet ranging from 12.5 to 19.23%. The evaluation study reflects the rigidity of the derived equations and their reliability in computing the best pipe-nozzle diameter ratios in discharge lines delivering free jets with reasonable powers, while the conventional formula is approximate. However, many engineering applications of water jet nozzles are used in; impulsive turbines, power-delivering free jets, water filters, flotation tanks, sedimentation tanks, water storage tanks, trickling filters, and other water and wastewater systems.

INTRODUCTION

The pipe-nozzle discharge lines are known for a wide range of applications in practice. They are generally used to having a high-velocity water jet that can be used for firefighting, mining, and power developments (the impulse turbines). Moreover, they are used in water filters, flotation tanks, sedimentation tanks, water storage tanks, trickling filters, units of water supply, and wastewater treatment stages. This is prevailing based on hydraulic considerations. In this case, the power of the jet is a function of its area and absolute velocity. To maximize its power, the nozzle opening has some intermediate size to achieve that (Streeter *et al.* 1985, SS Simon A.L. 1987, Somaida 1994, Finnemore and Joseph 2002). Analytically, this is an optimization problem and will be solved within the scope here using the derivative method associated with the hydraulic considerations and comparison requirements (Mazzoleni, 1994, Wright *et al.* 2003).

LITERATURE REVIEW

Very little research is found in the literature concerning the present problem. Most of the previous studies are directed toward having the maximum jet power delivered from the nozzle. They assume a constant coefficient of friction in the approach pipe and neglect minor losses caused by the nozzle (Streeter *et al.* 1985, SS Simon A.L. 1987, Somaida 1994, Finnemore and Joseph 2002). However, in the present study, the same problem is extended to introduce the hydraulic considerations to reach the best hydraulic pipe-nozzle diameter ratio in discharge lines delivering free jets, through the derivation of analytical solutions, evaluating derived equations, and comparison requirements. In practice, pipe nozzles are widely used in many water and wastewater engineering applications, such as irrigation systems, water supply, and wastewater system arrangements (Streeter *et al.* 1985, SS Simon A.L. 1987, Somaida 1994, Finnemore and Joseph 2002).

In 1994, a report about the basic properties of rotating, multi-jet, high-pressure water nozzles has been outlined. Orifice geometrics could produce highly compact jets (Mazzoleni, 1994). Regarding the nozzles' materials, no single nozzle material is suitable for all operating conditions. The selection of nozzle material should depend on how well the water will be filtered and on the operating pressure (Wright *et al.* 2003, Poeck, 2008, John *et al.* 2008 and Emmanuel at al. 2023).

The derivative method is reliable and effective to determine the optimum solution of many parameters in water systems. (Somaida *et al.* 2011) had a study to determine the optimal diameters of pipes in a network with a predetermined layout using computer simulation and analytical solutions for the optimum pipe diameters of water distribution pipe networks, supplied from a groundwater source by pumping. The optimization process is based on the derivative method and depends on the cost functions incorporating the various capital and operating costs (Somaida *et al.* 2011, (Somaida *et al.* 2012, (Somaida *et al.* 2013).

The nozzle design also has some effect on operating life;

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the initial jet quality produced is an indication of the quality of the nozzle design. For water jets used in sewer cleaning, the efficiency of the nozzles depends on the diameter of the nozzles and the pressure of a water jet at the outlet of the nozzle, and maximum efficiency appears for a pressure of 180 bar (Medan 2016). An optimizing nozzle design analytically was carried out, in which the optimization routine is a waterjet energy model, capable of resolving the abrasive energies for various nozzle geometries and operating conditions (Schwartzentruber *et al.* 2016). The process uses genetic algorithm optimization to vary the input parameters, to maximize the abrasive energies, the results show that the optimized nozzle designs could cut on average, 16% faster over a range of abrasive flow rates (Renjie *et al.* 2020, Radkevich 2021). Common applications of nozzles in water and wastewater systems can be summarized as follows: Evaporative disposal, such as disposal of excess water/chemical solution through evaporation, usually over a large pond. Foam control, spray nozzles to break up foam that can cause tank overflow, poor drainage, or other problems (ElZahar *et al.* 2001, De Cock 2017, Kawser *et al.* 2020, Amin and ElZahar 2022, ElZahar and Amin 2023). Tank mixing, and blending tank contents, homogenize sediment off the tank bottom to aid in transportation and filtration, sweeping solids across the bottom of the tank, and preventing thermal stratification. Filter nozzles, which can be installed in both open and closed filters, ensure maximum efficiency with minimum head losses (Amin and ElZahar 2022, ElZahar and Amin 2023, Fujisaki and ElZahar 2002, Amin and ElZahar 2023).

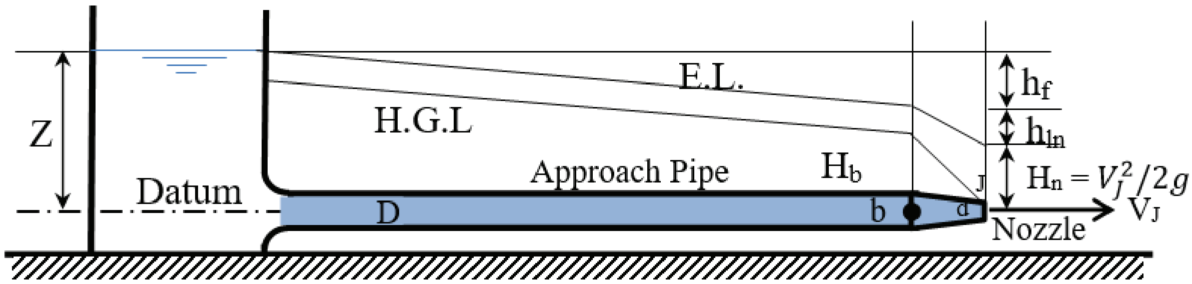

Figure 1: Pipe-nozzle discharge line arrangement

Best Pipe-Nozzle Diameter Ratios

Problem Formulation

In the pipe-nozzle discharge line shown in Fig. 1, the upstream head Z according to the energy equation given by, (John *et al.* 2011)

$Z=H_n+h_f+h_{ln}$ (1)

H_n = Kinetic head at the outlet of the nozzle.

h_f = head loss by friction in the approach pipe, and

h_{ln} = head loss due to energy loss through the nozzle.

It is noted that the entry loss is disregarded (well-rounded entrance) of the approach pipe. Substituting by , and in Eq. 1 and writing the equation for Hn, thus:

$H_n=Z-(FlV^2_p)/2gD+(1-C^2_v)H_b$ (2)

F = Coefficient of friction in the approach pipe,

L = Length of approach pipe,

g = Gravitational acceleration,

D = mean diameter of approach pipe,

V_p = Mean velocity of water in the approach pipe,

C_v = Coefficient of velocity in the nozzle, and

H_b = Net head at the base of the nozzle.

On the other hand, the power of the jet P_J is given by:

$P_J=\gamma QH_n$, where γ = specific weight of water, and Q = discharge from the nozzle, and $H_n=V^2_J/2g$,

(V_J = absolute velocity of the jet at the nozzle opening), then, $P_J=\gamma\ \pi/4\ \ d^2\ \ V_J\ \ H_n$ or

$P_J=Constant*\ (d^2 V_J)H_n$ (3)

Where, d = diameter of the nozzle opening.

For maximum power of the jet, the product $(d^2 V_J)H_n$ must be maximum, and in equation form will be:

$(d^2 V_J)H_n=maximum$ (4)

For P_J maximum, differentiate Eq. 4 with respect to d and equate to zero:

$d^2 V_J\ \ (\partial H_n)/\partial_d+2\ d\ V_j\ \ H_n=0$

or $(\partial H_n)/\partial d=-\ 2/d\ \ H_n=-\ 2/d\ \ ((V^2_J)/2g)$ (5)

From the continuity equation applied between base (b) and outlet of the nozzle (J), then $V_p=d^2/D^2\ \ V^J$.

Substitute in Eq. 2, then,

$H_n=Z-F\ l\ \ d^4/D^5\ \ \ (V^2_J)/2g-(1-C^2_v)\ H_b$ (6)

The efficiency of the nozzle η_n is given by, (Streeter *et al.* 1985)

$\eta_n=(V^2_J/2g)/H_b=C^2_v$

Where, C_v = Coefficient of velocity of the nozzle.

H_b=$(V^2_J)/(2g\ C^2_v)$ (7)

$H_n=Z-F\ l\ d^4/D^5\ (V^2_J)/2g-(1/(C^2_v)-1)\ (V^2_J)/2g$ (8)

Differentiate Eq. 8 with respect to d using variable pipe friction factor F and minor loss coefficient of the nozzle $(1/(C^2_v)-1)$ and steady flow conditions:

$(\partial H_n)/\partial d=-(l/D^5\ \partial F/\partial d)\ \ ((V^2_J/2g)-(\partial/\partial d)\ (1/(C^2_v)-1)$
$(V^2_J/2g)$ (9)

From equations 5 and 9, it is found that:

$2/d=(l/D^5\ \ \ (\partial F\ d^4)/\partial d)+\partial/\partial d\ (1/(C^2_v)-1)$ (10)

To evaluate the first partial in the R.H.S. of Eq. 10, $(\partial F\ \ d^4)/\partial d$, Von Karman formula for coefficient F, (Finnemore and Joseph 2002), can be used at rough, turbulent flow conditions in the approach pipe as follows:

$1/\sqrt{F}=2\ \log_{10}D/e+1.14$ (11)

Where e = roughness height of the pipe.

Introduce the parameter B which is known as the pipe-nozzle diameter ratio, (B=D/d), which is more than unity or D = B*d, then:

$1/\sqrt{F}=2 \log_{10}(B\ d)/e+1.14$ (12)
The term $(\partial F\ d^4)/\partial d$, in Eq. 10, can be differentiated as:
$(\partial F\ d^4)/\partial d=F*4d^3+d^4\ \partial F/\partial d$ (13)
The term $(\partial F)/\partial d$ can be evaluated from Eq. 12, as:
$\partial F/\partial d=-4\ F^{1.5}/d$ (14)
Substitute in (13), then: $(\partial F\ d^4)/\partial d=4\ F\ d^3 -4\ d^4\ \ F^{1.5}/d$
Or $(\partial F\ d^4)/\partial d=4\ d^3\ (F-F^{\wedge}1.5)$ (15)
In Eq. 10, the partial $\partial/\partial d\ (1/(C^2_v)-1)$ expresses the change in the loss coefficient of the nozzle with respect to its diameter. Since, the nozzle forms an integral part of the scheme, the above differential can be evaluated as follows:
For the flow in the nozzle, using the energy equation, the absolute velocity of the jet is given by, (Streeter *et al.* 1985):
$V_j=C_v\ \sqrt{(2\ g\ \Delta P/\gamma)/((1-(d/D)^4))}$ (16)
Where, $\Delta P/\gamma$ = change in the absolute pressure head between the base and outlet of the nozzle.
However, introducing the parameter C, known as the discharge coefficient of nozzle which is included in Eq. 17, (Streeter *et al.* 1985):
$C^2_v=C^2\ [1-(d/D)^4\ ]$ (17)
$\partial/\partial d\ (1/(C^2_v)-1)=\partial/\partial d\ (1/(C^2\ (1-1/B^4))-1)=(4/(C^2\ (1-1/B^4\ )^2\ B^5\ ))\partial B/\partial d-0$
As, $B=D/d$, $\partial B/\partial d=-D/d^2$, then,
$\partial/\partial d\ (1/(C^2_v)-1)=\ (4/(C^2\ (1-1/B^4)^2\ B^5\ ))\ \ \partial B/\partial d*-D/d^2$
Or $\partial/\partial d= -(4\ D)/(C^2\ d^2\ B^5\ (1-1/B^4\ )^2\ )$ (18)
Substituting by the partials (15) and (18) in Eq. 9, yields:
$2/d= l/D^5\ *4d^3*(F-F^{1.5}\ )-(4\ D)/C^2\ d^2\ B^5\ (1-1/B^4\ )^2\ )$
or $2/d=l/D*4/(D\ B^3)*(F-F^{1.5}\ )-(4\ B^4)/(C^2\ \ d^2)\ (B^4-1)^2)$(19)
Putting $d=D/B$ in Eq. 19, then,
$2B/d=l/D*4/(D\ B^3\ )*(F-F^{1.5})-(4\ B^5)/(C^2\ D\ (B^4-1)^2\ )$ (20)
Multiplying both sides of Eq. 14 by $(D\ B^3)/2$ and rearranging, then:
$B^4=2\ l/D\ (F-F^{1.5}\ )-(2\ B^8)/(C^2\ (B^4-1)^2\ )$ (21)
or $B^4+[(2\ B^8)/(C^2\ (B^4-1)^2\ )+2\ l/D\ F^{1.5}\ ]=2\ l/D\ F$ (22)
The term l/D in Eqs. 21 and 22, may be now called the relative distance. It must be noted that all the terms in Eq. 21 are dimensionless parameters. On the other hand, the well-known conventional equation for optimum pipe-nozzle diameter ratio is, (John *et al.* 2011):
$B^4=2\ \ l/D\ \ F$ (23)
Comparing Eqs. 22 and 23, it is evident that, when neglecting the terms between brackets in Eq. 22, concerning the changes of coefficient of friction F and the nozzle loss coefficient conditions with the nozzle diameter, then Eq. 22 reduces to Eq. 23.

Nozzle Flow Coefficient

From the experiments conducted on flow through nozzles at high Reynold's numbers (10^5 to less than 10^6) at different ratios D/d, the flow coefficients C are relatively constant, i.e., is a function of D/d only and is independent of Rn (Streeter *et al.* 1985, Finnemore and Joseph 2002).
According to (Finnemore and Joseph 2002), it is found practically that, the flow geometry is the same for flow of free jets from nozzles except for heads less than 3m (it will be taken 12m in the present study). However, an attempt is made by obtaining the flow nozzle coefficients (C) at values of D/d = 1.2-3.0, as shown in Table 1, being interpreted from, (Streeter *et al.* 1985).

Table 1: Flow coefficient C versus pipe-nozzle diameter ratio D/d (Streeter *et al.* 1985), Rn from 105-106

C	**D/d**
0.8919	1.2
0.8854	1.3
0.8794	1.4
0.8738	1.5
0.8686	1.6
0.8638	1.7
0.8593	1.8
0.8550	1.9
0.8510	2.0
0.8472	2.15
0.8337	2.25
0.8322	2.55
0.8198	3.0

However, the values of D/d and corresponding values of C are subjected to linear regression analysis and the produced equation has the following form:
$C=0.907/(D/d)^{0.092}$ (24)
Then, Eq. 24 is helpful in obtaining the flow coefficient C at a particular pipe-nozzle diameter ratio D/d during the present analysis.

Illustrative Example for Solving Equation 22

The application of Eq. 22 will be illustrated through the solution of the following illustrative example. The following data is given for a pipe-nozzle line discharging free jets, Fig. 1. Relative distance L/D=200, the upstream head Z is fixed at 12m. At each step of calculation, diameter of the approach pipe D=15 cm, relative roughness e/D=0.02 (rough pipe), and dynamic viscosity of water=1.13*10-3 N. Sec./m2 and discharge from the nozzle, Q=1000 lit./min. Evaluate F by Von Karman formula, $1/\sqrt{F}=2 \log_{10} 1/0.02+1.14$, then F=0.0486, can be taken safely, 0.05.
Calculating flow velocity in the approach pipe, $V_p= (4*1000)/(1000*60)*1/(\pi*0.15^2\ )=0.944$ m/sec.
Also, Reynold's number, $R_n=(1000*0.944*0.15)/1.13*10^3$ =125253, Rn>105, indicating rough, turbulent flow.
Eq. 22 will be solved by trial and error for values of D/d from 1.6-2.1, noting that at each diameter ratio, the flow coefficient C is calculated using Eq. 24. The results are illustrated in Table 2.

Table 2: Solution of Equation 22 by trial and error for D/d using the illustrative example data

D/d	C	Equation 22		Deviation %
		L.H.S.	**R.H.S.**	
1.6	0.8686	14.53	19.44	25.254
1.65	0.8662	15.26	19.44	21.503
1.7	0.8638	16.10	19.44	17.196
1.75	0.8615	17.04	19.44	12.340
1.8	0.8593	18.09	19.44	6.931
1.85	0.8571	19.25	19.44	0.959
1.855	0.8569	19.38	19.44	0.330
1.8575	0.8568	19.44	19.44	0.013
1.86	0.8567	19.50	19.44	0.305
1.87	0.8562	19.75	19.44	1.592
1.9	0.8550	20.53	19.44	5.593
1.95	0.8530	21.92	19.44	12.744
2	0.8510	23.43	19.44	20.515
2.1	0.8472	26.83	19.44	38.019

Investigation of Table 2, shows that, the optimum pipe-nozzle diameter ratio D/d is 1.8575, but it can be safely taken 1.9 with a negligible deviation of 0.022 (accepted). While the value of D/d using the well-known Eq. 23 is 2.1. The merit of one value of D/d over the other, can be determined through computing the maximum power of the jet from the nozzle in each case, as will be illustrated.

Table 3: Results for free jet power of pipe-nozzle diameter ratio D/d ranging from 1.1-3 (L/D = 200 and F=0.05)

D/d	C	C_v	Friction Loss Diameter FL d^4/D^5		V_J (m/sec)	V_P (m/sec)	Q (m^3/sec)	P_J (KW)
1.1	0.8991	0.5062	2.73	2.9027	4.6540	3.8463	0.0304	0.2230
1.2	0.8919	0.6418	1.93	1.4279	5.7000	3.9583	0.0310	0.5036
1.3	0.8854	0.7137	1.40	0.9630	6.5640	3.8840	0.0305	0.6570
1.4	0.8794	0.7563	1.04	0.7483	7.3320	3.7408	0.0294	0.7900
1.5	0.8738	0.7827	0.79	0.6321	8.0800	3.5911	0.0282	0.9200
1.6	0.8686	0.7996	0.61	0.5640	8.7300	3.4102	0.0268	1.0400
1.7	0.8638	0.8104	0.48	0.5226	9.3040	3.2194	0.0268	1.0940
1.8	0.8593	0.8173	0.38	0.4970	9.3600	2.8889	0.0253	1.1600
1.9	0.8550	0.8215	0.31	0.4817	10.3000	2.8532	0.0224	1.2000
2	0.8510	0.8239	0.25	0.4730	10.5310	2.6328	0.0207	1.1870
2.1	0.8472	0.8251	0.21	0.4689	10.8050	2.4501	0.0190	1.1670
2.5	0.8337	0.8229	0.10	0.4766	11.1000	1.7760	0.0172	1.0600
2.55	0.8322	0.8223	0.09	0.4791	11.5270	1.7727	0.0140	1.0130
3	0.8198	0.8147	0.05	0.5065	11.9000	1.3222	0.0130	0.7350

Merits of the Derived Equation 22

This can be shown when computing the maximum power of the jet at (=1.9) from Eq. 22, and at (2.1), from Eq. 23. This is made of an alternative computing procedure of jet power PJ as follows:

Determine, C, C_v, and C_v^2. Then determine the friction loss coefficient (and the minor loss coefficient, followed by computing the kinetic head at the outlet of the nozzle (, by solving Eq. 8 for V_J. Finally, determine V_P, Q, and P_J. The following data are also given, F = 0.05 and =200. The parameters computed are shown in Table 2. Also, a plot of versus power of the jet, P_J, is given in Fig. 3. Investigation of Table 3 as well as Fig. 2, shows that the power of the jet is attained at =1.9 and the power value is 1.2 KW, which is in good agreement with the value of, calculated using the derived Eq. 22, and Table 3. This indicates the rigidity of this equation. On the other hand, Eq. 23, gives = 2.1 at which the power of the jet P_J = 1.1

KW. Although both values of P_J have the same order of magnitude, the use of Eq. 22 has the merits of saving about 0.1 KW, i.e., there is an increase in the jet power of 9.1 %, over that produced by Eq. 23.

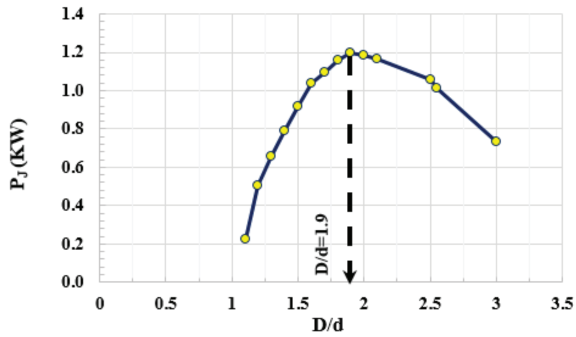

Figure 2: Correlation between results of D/d versus P_J

However, from the previous analysis, it is concluded that under a set of operational conditions, the derived Eq. 22, gives a unique optimal value for pipe-nozzle diameter at which the power of the jet is maximum.

A Simplified Form Of The Derived Equation

It is of practical interest to simplify Eq. 22, which is now put in the following form:

$[(2\,(D/d)^4)/(C^2\,(B^4-1)^2)+1]\;\;(D/d)^4=2\,l/D(F-F^{1.5})$ (25)

To do this, denote the L.H.S. of Eq. 25 by A, which is equal to:

$A=b\,(D/d)^4$ (26)

In which,

$b=[(2(D/d)^4)/(C^2(B^4-1)^2)+1]=[(2(B)^4)/(C^2(B^4-1)^2)+1]$(27)

Eq. 26 in the logarithmic form will be, $\ln A=\ln b+\ln(D/d)^4$, which is a straight-line equation of the form, Y=N+X. Where: Y= lnA, N= lnb, and X= $\ln(D/d)^4$. However, for diameter ratio D/d ranging from 1.4 to 3.584, determine C using Eq. 24 and the corresponding b by Eq. 27 followed by: A, Y= lnA, and X= $\ln(D/d)^4$, at each value of D/d.

These parameters are shown in Table 3 and are needed for linear regression analysis of the left-hand side of Eq. 25. Computations according to (Streeter *et al.* 1985), pp. 400-451, show that:

$N=(\sum Y_i-m\sum X_i)/n=(59.15752-1*55.26881)/18=0.216039=\ln b$, from which b = 1.24115 and the developed equation will have the form:

$(D/d)^4=1/1.24115\;[2\,l/D\;(F-F^{1.5})]$

or, $(D/d)^4=1/1.24115\;[2\,l/D\;(F-F^{1.5})]$

$D/d=0.95\;[2\,l/D(F-F^{1.5})]^{0.25}$ (28)

To check the applicability of Eq. 28, substitute by D/d=200, and F=0.05 from the illustrative example, then D/d= 0.95 [2*200*(0.05-0.051.5)]0.25=1.8858, which is closely equal to the value of D/d=1.9 reached by Eq. 22. The deviation is 0.745%, which is fully accepted.

However, Eq. 28 holds good for diameter ratio D/d from 1.4 to 3.584. It is evident that: (A) Eq. 28, which is the simpler form of Eq. 22, can be applied without

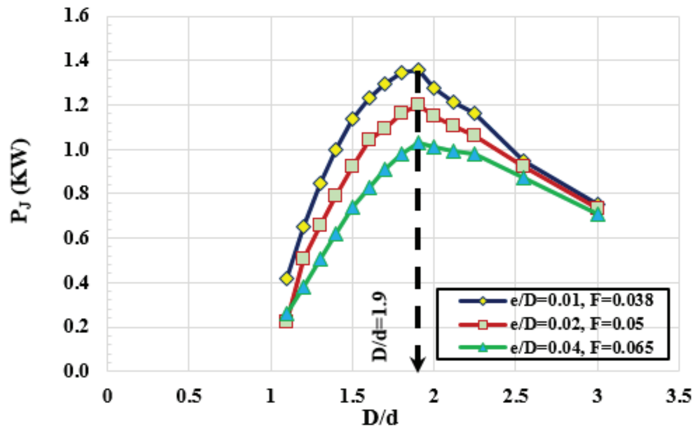

Figure 3: Plots of D/d versus P_J for illustrative example

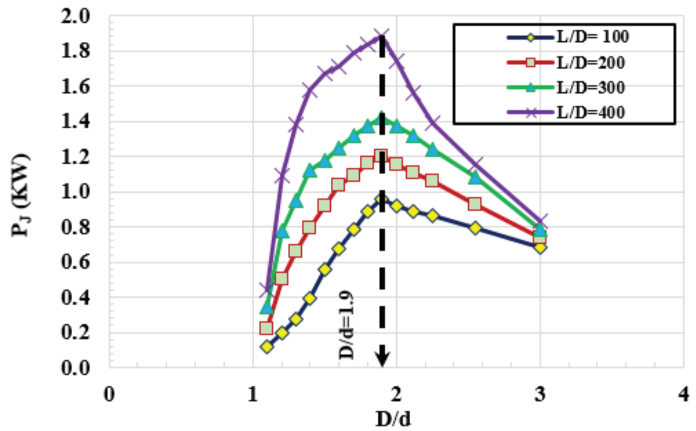

Figure 4: Plots of D/d versus P_J, at different L/D, F=0.05 for illustrative example

the need for use of computer programs. (B) The best hydraulic pipe-nozzle diameter ratios D/d are function of relative distance L/D and coefficient of friction F. (C) The errors produced in computing D/d are negligible. (D) Neglecting the changes of F and C_v, Eq. 27 is reduced to the well-known Eq. 23, $D/d=[2\ l/D\ \ F]^{0.25}$. This may indicate the rigidity of the derived equations for pipe-nozzle diameter ratio.

Evaluation Study Of The Derived Equation

Variation of Jet Power PJ and Ratio D/d

Using the data of the illustrative example, the required parameters for this evaluation are computed and illustrated in Tables 5, 6, and 7, and their plots are given in Fig. 3. Investigation of the plots leads to the following conclusions:

(A) The plots of D/d versus P_J, exhibit similar trends at various of e/D.

(B) The maximum P_J is indicated at D/d= 1.9, indicating the validity of the derived Eq. 22.

(C) For D/d > 1.9, P_J drops. This is attributed to the lesser jet area relative to the approach pipe as well as lesser Q regardless of the higher jet velocity V_J attained.

(D) For D/d<1.9, P_J also drops, due to the larger area of the jet relative to the approach pipe, larger Q, and V_P in the pipe leading to an increase of friction and energy losses at the expense of P_J. The best P_J is achieved at D/d = 1.9 and e/D = 0.01 (F=0.038), which equals 1.36 KW, this is due to lesser friction and minor losses.

Table 4: Data required for linear regression analysis of L.H.S. of equation 22

D/d	B^4	C	C_v	b	Ln (b)	Ln $(D/d)^4$	A	Ln (A)
1.5	5.0625	0.8738	0.7827	1.803516	0.589738	1.62186	9.130299	2.211598
1.6	6.5536	0.8686	0.7996	1.563254	0.44677	1.880015	10.24494	2.326784
1.7	8.3521	0.8638	0.8104	1.414182	0.346551	2.122513	11.81139	2.469064
1.8	10.4976	0.8593	0.8173	1.315245	0.274023	2.351147	13.80691	2.625169
1.9	13.0321	0.8550	0.8215	1.246285	0.220167	2.567416	16.24171	2.787583
1.97	15.06138	0.8522	0.8234	1.2098	0.190455	2.712134	18.22127	2.902589
2	16	0.8510	0.8239	1.1964	0.179317	2.772589	19.14241	2.951906
2.01	16.32241	0.8506	0.8241	1.192191	0.175793	2.792539	19.45943	2.968332
2.1	19.4481	0.8472	0.8251	1.159249	0.147773	2.967749	22.5452	3.115522
2.174	22.33768	0.8445	0.8253	1.137599	0.12892	3.106275	25.41132	3.235195
2.25	25.62891	0.8418	0.8252	1.11925	0.112658	3.243721	28.68514	3.356379
2.558	42.81561	0.8319	0.8221	1.070761	0.06837	3.756903	45.84529	3.825272
2.565	43.2862	0.8317	0.8220	1.069991	0.06765	3.767834	46.31583	3.835484
2.667	50.59319	0.8287	0.8205	1.059903	0.058178	3.923817	53.6239	3.981995
2.985	79.39211	0.8202	0.8150	1.038409	0.03769	4.374399	82.44151	4.412089
3.367	128.5205	0.8111	0.8080	1.024024	0.02374	4.856089	131.6081	4.879828
3.584	164.9955	0.8065	0.8041	1.018864	0.018688	5.105918	168.1079	5.124606
						55.26881		59.15752

Variation of Jet Power P_J and Ratio L/D

The parameters D/d, P_J, Rn are computed at different relative distances L/D (50, 100, 200, 300, and 400), e/D = 0.02 (F = 0.05), and Z = 12m. The results are illustrated in Table 8 and Fig. 4. These indicate that: (A) All the plots exhibit similar trends. (B) As L/D increases, PJ increases and vice versa, the rate of increase is uniformly slight between L/D= 200 and 300, due to constant energy losses. (C) The plots show an optimum unique value of D/d=1.9 at which PJ max.=1.2 KW.

Table 5: Results for free jet power P_J (D/d=1-3, e/D=0.01, F=0.038, L/D=200)

D/d	C	C_v	B^4	FLd^4/D^5		V_J(m/sec)	V_p(m/sec)	Q(m^3/sec)	P_J (KW)	R_n
1.1	0.8991	0.506	1.464	5.1919	3.9027	5.088	4.205	0.0330	0.420	372124
1.2	0.8919	0.642	2.074	3.665	2.4279	6.216	4.318	0.0339	0.650	382018
1.3	0.8854	0.714	2.856	2.661	1.9630	7.136	4.222	0.0331	0.844	373646
1.4	0.8794	0.756	3.842	1.978	1.7483	7.919	4.04	0.0317	1.000	357522
1.5	0.8738	0.783	5.063	1.501	1.6321	8.669	3.853	0.0302	1.136	340973
1.6	0.8686	0.799	6.554	1.160	1.5640	9.297	3.632	0.0285	1.232	321398
1.7	0.8638	0.810	8.352	0.910	1.5226	9.838	3.404	0.0267	1.293	301239
1.8	0.8593	0.817	10.498	0.724	1.4970	10.361	3.2	0.0251	1.347	283186

1.9	0.8550	0.821	13.032	0.583	1.4817	10.770	2.982	0.0234	1.360	263894
2	0.8510	0.824	16	0.475	1.4730	10.920	2.731	0.0214	1.278	241681
2.115	0.8466	0.825	20.010	0.380	1.4686	11.142	2.49	0.0196	1.214	220354
2.25	0.8418	0.825	25.629	0.297	1.4685	11.440	2.26	0.0177	1.160	200000
2.55	0.8322	0.822	42.283	0.180	1.4790	11.635	1.79	0.0141	0.950	158402
3	0.8198	0.815	81	0.094	1.5065	12.007	1.334	0.0105	0.750	118053

Comparison between the Derived and Conventional Equation

Variation of P_J and L/D

The results of P_J and L/D are given in Table 8 and the plots are shown in Fig. 5, which indicates the following: (A) In Eq. 22, P_J increases with the increase of L/D. (B) In the conventional Eq. 23, P_J decreases slightly at a uniform rate. (C) On the increase of L/D, the two plots converge, intersect, diverge, and then converge again. This may be due to the nature of both equations; both reflect the effects of L/D and P_J.

Variation of D/d and R_n

The results of D/d and Rn are given in Table 8. The corresponding plot is shown in Fig. 7. Note that Rn is computed for the approach pipe rather than the opening of the nozzle. In both plots, at lower values of Rn, D/d is increased. This is logical, i.e., pipe velocity is decreased, however, the diameter of the approach pipe is increased relative to the diameter of the nozzle. The opposite sides of both plots may be due to the nature of both equations, as both curves become similarly diverging. Coincident trends are observed till Rn is about 260000, after which both curves become diverging.

Table 6: Results for free jet power P_J (D/d=1-3, e/D=0.02, F=0.05, L/D=200)

D/d	C	C_v	B^4	FLd^4/D^5		V_J (m/sec)	V_P (m/sec)	Q (m^3/sec)	P_J (KW)	R_n
1.1	0.8991	0.5062	1.4641	6.8301	3.90266	4.684	3.87	0.0304	0.223	342248
1.2	0.8919	0.6418	2.0736	4.8225	2.42794	5.700	3.96	0.0310	0.504	350442
1.3	0.8854	0.7137	2.8561	3.5013	1.96301	5.564	3.884	0.0305	0.657	343372
1.4	0.8794	0.7563	3.8416	2.6031	1.74833	7.332	3.74	0.0294	0.790	330973
1.5	0.8738	0.7827	5.0625	1.9753	1.63214	8.079	3.59	0.0282	0.920	317700
1.6	0.8686	0.7996	6.5536	1.5259	1.56404	8.729	3.41	0.0268	1.040	301177
1.7	0.8638	0.8104	8.3521	1.1973	1.52255	9.304	3.219	0.0259	1.094	284490
1.8	0.8593	0.8173	10.4976	0.9526	1.49703	9.860	3.043	0.0240	1.160	269292
1.9	0.8550	0.8215	13.0321	0.7673	1.48166	10.310	2.856	0.0224	1.200	252274
2	0.8510	0.8239	16	0.6250	1.473	10.530	2.6336	0.0267	1.150	232973
2.115	0.8466	0.8252	20.0097	0.4998	1.46861	10.808	2.4165	0.0190	1.107	213761
2.25	0.8418	0.8252	25.6289	0.3902	1.46849	11.100	2.193	0.0172	1.060	194070
2.55	0.8322	0.8223	42.2825	0.2365	1.47905	11.572	1.7727	0.0139	0.925	156876
3	0.8198	0.8147	81	0.1235	1.50651	11.900	1.3221	0.0104	0.735	117003

Variation of P_J and Reynold's Number R_n

The results of the necessary parameters as shown in Table 8, and their plots in Fig. 7, indicate that: (A) As the relative distance L/D increases, D/d increases in both equations, Table 8 because D/d is a function of L/D. (B) The conventional equation shows the increase of P_J with increasing R_n, while in the derived Eq. 28, P_J increases till

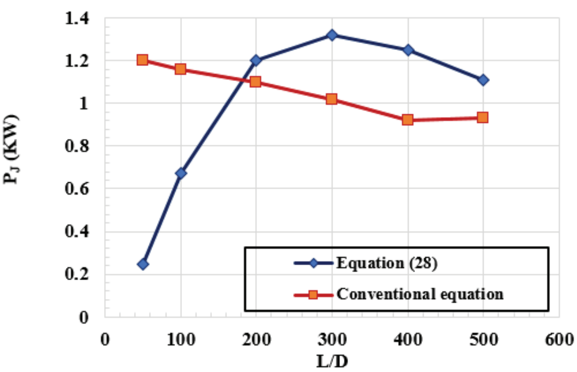

Figure 5: Plots of P_J versus L/D using the derived Equation 28 and the conventional one, (e/D = 0.02, F = 0.05)

Rn is about 226000 (about 1.2 KW), then it drops rapidly till reaching a value of 0.25 KW at R_n just below 290000, Table 8, and Fig. 7. This is attributed to the nature of both equations. (C) However, it may be stated that Eq. 28 is advantageous when applied at Rn between 150000 to 250000, because it attains P_J values higher than the conventional formula (23), with relative distance L/D in the range (50-500).

Table 7: Results for free jet power P_J (D/d=1-3, e/D=0.04, F=0.065, L/D=200)

D/d	C	C_v	B^4	FLd^4/D^5		V_J (m/sec)	V_P (m/sec)	Q (m^3/sec)	P_J (KW)	R_n
1.1	0.8991	0.5062	1.4641	8.8792	3.90266	4.292	3.547	0.0278	0.26	313894
1.2	0.8919	0.6418	2.0736	6.2693	2.42794	5.200	3.613	0.0254	0.38	319735
1.3	0.8854	0.7137	2.8561	4.5517	1.96301	6.011	3.557	0.0280	0.504	314779
1.4	0.8794	0.7563	3.8416	3.3840	1.74833	6.755	3.446	0.0270	0.62	304956
1.5	0.8738	0.7827	5.0625	2.5679	1.63214	7.520	3.34	0.0262	0.74	295575
1.6	0.8686	0.7996	6.5536	1.9836	1.56404	8.150	3.184	0.0250	0.83	281770
1.7	0.8638	0.8104	8.3521	1.5565	1.52255	8.745	3.026	0.0238	0.91	267788
1.8	0.8593	0.8173	10.4976	1.2384	1.49703	9.325	2.88	0.0226	0.982	254867
1.9	0.8550	0.8215	13.0321	0.9975	1.48166	9.812	2.718	0.0213	1.027	240531
2	0.8510	0.8239	16	0.8125	1.473	10.100	2.5235	0.0200	1.01	223319
2.115	0.8466	0.8252	20.0097	0.6497	1.46861	10.425	2.3305	0.0183	0.994	206239
2.25	0.8418	0.8252	25.6289	0.5072	1.46849	10.824	2.138	0.0168	0.98	189204
2.55	0.8322	0.8223	42.2825	0.3075	1.47905	11.300	1.7382	0.0136	0.87	153787
3	0.8198	0.8147	81	0.1605	1.50651	11.769	1.3077	0.0126	0.711	115726

Table 8: Results of D/d, P_J, and R_n computed using derived Eq. 28 and the conventional one, (Z=12m, F=0.05, L/D=50-500)

Eq. 28				**Conventional equation**		
L/D	**D/d**	**P_J (KW)**	**R_n**	**D/d**	**P_J (KW)**	**R_n**
50	1.333	0.246	287788	1.5	1.2	328710
100	1.586	0.67	261947	1.75	1.16	273215
200	1.886	1.2	252274	2.115	1.1	213317
300	2.087	1.32	226885	2.34	1.02	181754
400	2.243	1.25	202422	2.5	0.92	160812
500	2.371	1.11	166726	2.66	0.93	143051

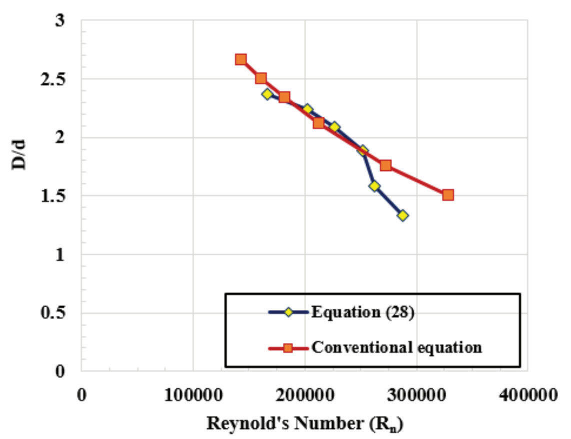

Figure 6: Plots of D/d versus Reynold's number using the derived Equation 28 and the conventional one, (e/D = 0.02, F = 0.05, L/D=50-500)

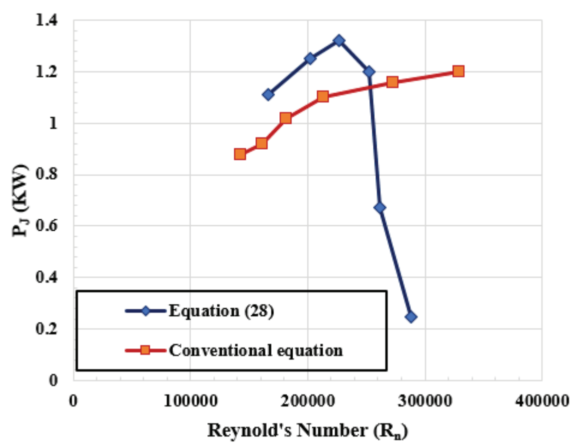

Figure 7: Plots of P_J versus Reynold's number using the derived Equation 28 and the conventional one, (e/D = 0.02, F = 0.05, L/D=50-500)

CONCLUSION

According to the present study, the conclusions reached could be summarized as follows:

(1) The derived equations for computing the best pipe-nozzle diameter ratio D/d in discharge lines delivering free jets, are applicable over practical ranges of relative distance L/D and relative roughness e/D under rough, turbulent flow in approach pipe. For the time being, L/D ranges from 50 to 500, e/D ranges from 0.01 to 0.05, and Rn ranges from 10^5 to less than 10^6.

(2) For a given illustrative example, Eq. 22 derived for D/d, is solved by trial and error and shows that the optimum D/d is closely equal to 1.9 and maximizes the jet power to 1.2 KW. At D/d=1.8. P_J = 1.16 KW and at D/d=2.1, is 1.1 KW.

(3) Also, Eq. 28, which is the simplest form of Eq. 22 being reached by linear regression analysis and applied to the same illustrative example, gives D/d=1.8858 (closely equal to 1.9), the deviation being 0.77%, which is negligible. However, Eqs. 22 and 28, hold good for D/d from 1.4 to 3.584 and could be applied to compute D/d ratios without the need for a computer program.

(4) Investigation of the derived equations, the diameter ratio D/d is a function of relative distance L/D and friction factor F, and computational errors are negligible.

(5) In the derived Eq. 22, if the change of coefficient of friction F (approach pipe), as well as the minor losses due to the nozzle, are neglected, the equation is reduced to the well-known conventional equation for D/d which can be used for a rough estimation of the approach pipe nozzle diameter ratio D/d.

(6) In both derived Eqs. 22 and 28, D/d increases with the decrease of Reynold's number R_n. This is due to the lower pipe velocity and larger diameter of the approach pipe compared with the jet diameter.

(7) The conventional formula gives lower jet power P_J at R_n<250000, while in this range, Eqs. 22 or 28 attain larger values of P_J and the increase in the computed P_J ranges from 12.5 to 19.2 %.

However, it may be stated that, the evaluation of Eqs. 22 and 28 reflect the rigidity of these equations and their reliability in computing the best pipe-nozzle diameter ratios in discharge lines delivering free jets with reasonable powers, while the conventional formula is considered approximate.

Nomenclature

C = Flow ratio of the nozzle,
C_v = Coefficient of the velocity of the nozzle,
D = Mean diameter of approach pipe,
d = Diameter of the nozzle opening,
e = Roughness height in approach pipe,
e/D = Relative roughness,
F = Coefficient of friction in the approach pipe,
g = Gravitational acceleration,
H_n = Kinetic head at the outlet of the nozzle,
h_f = Head loss by friction in the approach pipe,
h_{ln} = Head loss due to energy losses in the nozzle,
H_b = Net head at the base of the nozzle,
L = Length of approach pipe,
L/D = Relative distance,
P_J = Power of the jet,
 = Change in the absolute pressure head between the base and outlet of the nozzle,
Q = Discharge from the nozzle,
V_J = Absolute velocity of the jet at the nozzle opening,
V_p = Mean velocity of water in the approach pipe,
Z = Upstream head,
 = Specific weight of water,
η = Efficiency of the nozzle.

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