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Learner Centred Teaching Strategies That Can Be Incorporated with Guided Discovery Method on Senior High School Students' Achievement in Circle Theorems

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ABSTRACT

The aim of the study is to investigate learner centered teaching strategies that can be incorporated with the guided discovery method to aid students' achievement in circle theorems. The study employed a descriptive survey of questionnaires formulated for both teachers and students. Two Form 2 classes from various two Wa Municipality schools were chosen for the study using convenient, purposeful, and straightforward random sampling techniques. To conduct the study, two research questions were used. The sample size was composed of 164 students and 32 mathematics teachers from 2 schools in Wa Municipality in the Upper West Region. The theory that influenced the study was Piaget's cognitive constructivism. The main purpose of the study was to ascertain the impact of incorporating other teaching or learning strategies with the guided discovery-based instruction to aid students' conceptual understanding leading to achievement in circle theorems. The study's findings indicated that teacher-related challenges recorded a mean of 1.66 as well as students' own personal problems recorded a mean of 1.71, the encounter with teaching and learning of circle theorems had a low effect rate of 1.71 on their understanding of circle theorems. The results of the study's findings suggested that teachers' implementation of a variety of other appropriate learner-centered teaching strategies recorded a mean of 1.68 as well and learner-centered teaching resources or teaching aids recorded a mean of 1.77, have a low effect rate on students' understanding of circle theorems when they are incorporated with guided discovery method in the classroom. To enhance the performance of SHS students, it will be most advantageous to introduce guided discovery teaching methods to pre-and in-service teachers, through promotion by Ghana Education Service and/or other stakeholders in the education sector, this introduction may be distributed through workshops and seminars. Math teachers' instructional techniques and skills will improve as a result.

INTRODUCTION

One of the fundamental courses that is extremely beneficial to society and requires a lot of focus in order to meet human needs is mathematics (Charles-Ogun, 2015). The primary benefit of mathematics is needed in both social and economic transformation of every particular nation due to the visibility nature of it (Charles-Ogun, 2015). This is to imply that every nation needs technology as an interventional tool for development and one cannot obtain achievement without knowledge of technology. In touting the significance of mathematics education in Einstein (1934), stated that "Mathematics contains the real creative principle in physics" (Referenced on page 6 of Karam & Pospiech, 2010). This asserts to the inter dependency of other courses like physics relying on the knowledge of mathematics for development. The right concepts and procedures must be developed towards for understanding of topics for instance, geometry, algebra, calculus, discrete mathematics and probability. The study of space, including plane shapes, solid shapes, and relationships between plane surfaces, is the area of mathematics that geometry focuses on. "The Elements," Euclid's geometry textbook, is where he first introduced the mathematical system known as "Euclidean geometry." (Steyn, 2016 p.7). The significance nature of mathematics is credited as a measurement of civilization that has

made it more conditional to anyone that desire to pursue education in the fields of science, technology, and other pertinent fields of study Avong, (2013). For a successful development of life is due to the contribution of science and mathematics instruction.

As a result, the researcher found that the majority of students taught the circle theorem was a challenging topic. Those who answer questions on this topic on rare cases exhibited a lack of comprehension of the subject or the issue has been raised in studies proposed by, (Fletcher & Anderson, 2012). Additionally, the Council of Chief Examiners for the West African Examinations has reported WAEC, 2011, 2012, 2014, 2015, and (2018), are included, and they revealed that there were many students who performed poorly in their understanding of the circle theorem concepts when compared to other areas of mathematics. A study by Boson-Amedenu (2017), found that the senior high school studies of basic mathematics curriculum and syllables contain many perceived "difficult concepts."

"Exposure to construct or reconstruct mathematical concepts helps students learn mathematics," researchers have found. This means that math's educators need to reconsider their current methods of instruction Khanal (2018), page 6. Numerous research has pushed for the inclusion of learner-friendly teaching and learning

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techniques in mathematics instructors' studies of geometry Hou, (2007). Obtaining learning objectives can be done in a few different ways. According to Haeruman, Rahayu, and Ambarwati (2017), learning objectives can be met by the use of teaching tactics that are specifically suited to the needs and aptitudes of each individual student. This will make the learning material feel inviting and worthwhile.

The implementation of learner centered teaching strategies and learning models needs to pay attention to the characteristics and conditions of students so that their implementation is effective. The application of learning models and strategies affects the level of student characteristics as the condition of the students being studied. A lot of Ghanaian Senior Secondary School students could not solve problems of Euclidean circle geometrical construction due to their inadequate grasp of the ideas pertaining to fundamental circle theorem knowledge Boson-Amedenu (2017), Hissan & Ntow (2021), (Dogwi, 2014; Lim, 1992). Guided discovery Learning is a learning model that encourages students to discover concept by themselves (Jasri & Masunah, 2019) (Faradillah, *et al.*, 2021). The teacher's presentation of lesson materials is not final, but students are allowed to find their own through a problem-solving approach technique. Research that is relevant to research on the development of mathematics learning tools based on guided discoveries is the research of Yerizon *et al.* (2018), on the development of guided discovery-based learning tools to improve problem-solving skills. Based on several national and international studies relevant to this research, both research on mathematics learning. In general, the research uses a guided discovery model but there are several using different models; Dewi (2016) uses the development of problem-based learning tools to improve problem-solving ability.

At the same time, there is also something relevant to the ability used, namely the ability to solve mathematics problems. In general, guided discovery model-based learning tools can improve mathematics problem-solving skills in junior high school grade VII. The relevant research is only used as a basis for developing and enhancing the research to be carried out. Researchers hope this mathematics learning tool will later produce a valid, practical, and effective mathematics learning tool. The researcher therefore seeks to investigate learner centered teaching strategies that can be incorporated with guided discovery-based instruction with efforts of mathematics teachers to ensure efficient implementation to aid students' conceptual understanding leading to

students' achievement in circle theorems on the various Senior High Schools in the Municipality.

Statement of the Problem

The Chief Examiner for Core Mathematics Reports identifies weaknesses in mathematics teaching due to students' shallow understanding of circle theorems concepts. (WAEC, 2013, 2016). The West African Examination Council (WAEC), 2013, and 2016 reports of the Chief Examiner for Core Mathematics in Ghana indicate that a sizable portion of students have demonstrable weaknesses in answering questions involving the use of circle theorems.

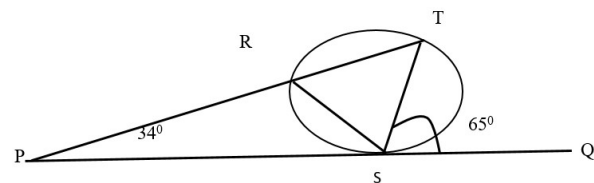


Figure 1: Diagram B WASSCE May/June 2017 Question 9

Most of the time, students shy away from inquiries that involve applying circle theorems. For instance, the Chief Examiner's reports for Core Mathematics for the May/June 2013 and 2016 exams showed that the majority of candidates did not even attempt to solve questions 10(a) and 11(b), which required the use of geometrical concepts. The few applicants who attempted these questions reportedly struggled, exhibiting inadequate subject knowledge and the application of geometric theorems (WAEC, 2017). Additionally, as shown in Diagram A below, the majority of candidates were unable to use the proper geometrical theorems to calculate the value of some given angles in Question 9 according to the Chief Examiner's report for Core Mathematics in 2017. Candidates had to draw a circle RST, a tangent PQ to the circle, and then a straight line PRT as seen in the diagram A. The report claims that the majority of applicants found this question to be very unpopular, and the few applicants who attempted it were unable to draw the diagram necessary to calculate the necessary angles, proving their lack of proficiency in geometry.

WASSCE May/ June 2017. Question 9

The report claims that most candidates struggled conceptually when drawing the necessary diagram, which resulted in generally subpar performance on this question (required diagram reported in diagram B) (WAEC, 2013, 2016). The Chief Examiner also notes that most

At point S, PQ is tangent to the SRT curve. PRT has the following properties: $\angle TPS = 34^\circ$, and $\angle TSQ = 65^\circ$.

1. Demonstrate the given information in a diagram.
2. Determine the value of
 - a) $\angle RTS$ and b) $\angle SRP$.

Figure 2: Diagram A for Question 9 of the May/June 2017 WASSCE

candidates had trouble understanding questions that included, among other concepts, angles on the outside and inside of polygons, as well as their characteristics (WAEC, 2017; WAEC & GOG, 2019). The researcher observed a low student proficiency in plane geometry and mensuration, from WASSCE 2016 through WASSCE 2022 particularly in circle theorems during coordination. The word “geometry,” say Johnson-Wilder and Mason (2005), scares the average primary teacher. As a result, encouraging the teaching of geometry in any form seems difficult in primary schools, and some primary teachers’ textbooks give it scant attention. Students in Ghana struggle with geometry and circle theorems due to teacher-dominated teaching methods. Primary schools focus more on arithmetic, leading to negative opinions of mathematics in senior high school (Tay & Mensah-Wonkyi, 2018). The students’ poor geometry performance and lack of knowledge may be due to teachers’ poor mathematical backgrounds, making achieving future goals difficult (Wilmot, Yarkwah, & Abreh, 2018). The author is considering further study on an effective teaching strategy for SHS students, considering the Core Mathematics Teaching Syllabus, to ensure even weak students make progress. (CRDD, Ministry of Education, 2010), Plane Geometry 2, which is covered in SHS2, includes the topic of circle geometry. Students should succeed in the following vital particular objectives in Plane Geometry 2:

1. Study, explain, and apply circle theorems
2. Determine the point’s radius must be perpendicular to the tangent of contact.
3. Check to see if the angle matches the angle in the alternate segment between the chord and tangent at the point of contact.

The WASSCE circle theorem performance of Ghanaian students is still below average (WAEC, 2017, 2018). Circle theorems are crucial in geometric study, requiring efficient learner centered teaching strategies incorporating onto guided discovery-based instruction. This approach improves students’ mathematical problem-solving abilities through learner-centered methods (Incikabi & Kara, 2018). A study suggests that using GeoGebra as an ICT tool which is a learner centered teaching activity in guided discovery learning can enhance students’ mathematical comprehension and problem-solving skills (Sariyasa, Ardana, & Murni, 2017). Studies by Yismaw and Gurju (2018), address the challenges and interest needs of students’ learning capabilities. This application software empowers students to generate their own knowledge through guided discovery-based methods, enabling them to tackle global problems (Bray & Tangney, 2017).

Learner centred teaching activities such as using board diagrams and mathematical games, can be incorporated with guided discovery methods, to aid students in understanding and facilitating effective learning within the school environment. The guided discovery approach in geometry teaching utilizes diagrams to illustrate concepts, aiding communication and identifying common identities

and properties between diagrams during the study of Schwartz and Heiser (2006), noticed that the diagrams’ illustration in concepts of circle theorems foster a sense of spatial reasoning among students in order to achieve knowledge in learning it. The study emphasizes the importance of visualizing geometrical drawings in teaching circle theorems. The study of Herbst, Gonzalez, and Macke (2005), agreed that the aim of investigating the preparation of teachers towards how they may use students on the ground to define a figure in a meaningful way. Computer-assisted instruction as child method can be incorporated into discovery learning strategies enhance student-learning outcomes, with scaffolding and discovery learning achieving minimum completeness criteria, with exploration-based education statistics showing superior performance (Nugroho & Hidayah, 2020). Allowing students to find their own information in the form of a concept, rule, or theorem is part of the teacher-guided discovery method used in the learning process (Mardati, 2018).

The researcher observed students’ poor performance in circle theorems at Wa Municipality High School from 2015-2022 due to shallow understanding, suggesting improved instruction through guided discovery, but no studies on this topic exist. The effectiveness or legitimacy of guided discovery teaching strategies in educational institutions has been assessed by studies (Ramadhani, 2017), (Ozomadu’s, 2016), (Simamora, & Saragih, 2019), (Nugroho, & Hidayah, 2020). They appear to have covered many topics, but not circle theorem geometry. Additionally, they developed guided discovery teaching strategies as a pedagogical strategy (Pujiastuti, Kusumah, Sumarmo, and Dahlan, 2014), (Arynda, & Susanto, 2012), leaving remedies that could be employed to solve concepts of circle theorems by using guided discovery teaching method on students’ conceptual understanding of circle theorems as well as challenges affecting students’ achievement in teaching and learning of circle theorems. The gaps found serve as the primary focus of this study.

Purpose of the Study

The main purpose of the study was to investigate into learner centered-teaching strategies that can be incorporated with guided discovery-based instruction to aid students’ conceptual understanding leading to achievement in circle theorems.

Research Question

1. What are the challenges affecting students’ achievement in teaching and learning circle theorems among SHS in Wa Municipality?
2. What remedies can be employed in solving concepts of circle theorems using guided discovery teaching methods on students’ achievement among SHS in Wa Municipality?

LITERATURE REVIEW

Theoretical Framework

Piaget’s idea of cognitive constructivism was created

by adapting human knowledge without the ability to learn something about the outside world from our own experiences (Von Glasersfeld, 1995). Regarding research done by Phillips and Soltis (2004, p. 7), it is generally recognized that knowledge is suggested for organizing experience with the process of “Piaget considered learning as an organism’s adaptive function. Teachers use various learner centred teaching methods, such as diagram illustrations, mathematical instruments, role-playing, brainstorming, group activities, and ICT tools like GeoGebra, which can be incorporated with guided discovery method to teach Euclidean Geometry theorems. Piaget’s concept of learning involves the construction and modification of structures for successful navigation. The guided discovery method, which involves students drawing the eight theorems of Euclidean geometry using tools like compasses, protractors, dividers, and instructional sheet, helps learners visualize concepts and develop new ideas. This approach helps students understand the usefulness of geometry in their lives and develops new mathematical concepts. Piaget’s studies suggest that humans cannot easily understand new information and knowledge construction is crucial through individual efforts. (Piaget, 1953). Knowledge is built through assimilation and accommodation in developmental stages, such as sensorimotor, concrete, formal, and preoperational stages. Children learn Euclidean Geometry through cognitive conflict and constructivism, leading to expertise in learning. The guided discovery method, which involves students performing mathematical activities, helps maintain equilibrium and promotes critical thinking. This method is superior to direct teacher learning, as it allows learners to acquire knowledge, skills, and optimistic attitudes through hands-on experiences. This approach helps students develop critical thinking skills and solve math problems, ultimately improving their overall learning experience. This is in line with the findings of Jumhariyani’s (2016) study, which used the guided discovery teaching method to discover a link between how students learn and their capacity for critical thought when it comes to mathematics.

Challenges Teachers Face Towards Teaching Circle Geometry That Affect Students’ Achievement in Concepts of Circle Theorems

Demonstrations of characteristics or features of a highly qualified teacher possess to conduct effective teaching and learning of mathematics Stronge, (2018), according to the statement on page 15, teaching “requires teachers to possess substantial knowledge base, which encompasses subject matter knowledge, pedagogical knowledge, curricular knowledge, learner knowledge, and cultural and community knowledge” Moreover, in the learning institutions if such attributes are able to be implemented by a teacher will help to acquire academic achievement and yield good positive results towards the study of mathematical concepts in Euclidean geometry. Research reveals that, “Silence on a team can be a sign

that someone is drowning. Or it can be a canary in the coal mine” (Ludema & Johnson, 2019 p. 1). So therefore, teachers developed the attitude of silence that may either be a negative or positive behaviour towards meetings in the school will affect academic performance leading to low rate of achievement. According to literature silence can indicates that an individual is struggling or issues that are systematic and need a great attention for the general health of the entire school community to developed (Mavrogordato & White, 2020). Behaviours such as oppression and discrimination manifest by individuals during meetings to maintain silence goes to affect the conduct of teaching in mathematics and many more courses (Sherf, Parke, & Isaakyan, 2021).

In terms of behaviour and perceptions of understanding students’ mathematical thinking abilities, a study shows differences between expert and non-expert teachers in mathematics education (Yu, Cai, & Zhu, 2018). A secondary mathematics teacher “may believe statistics is difficult and this may impact their perceptions about their ability to teach statistics content” (p. 300; Lovett & Lee, 2017), according to a study of pre-service teachers. Additionally, researchers like Hedges and Harkness (2017), concurred that students’ perspectives or opinions can be categorized into one of three ways: “(1) Statistics is Mathematics, (2) Statistics is like Geometry, and also (3) Statistics is almost not Mathematics” (p. 349). “The inability of mathematics teachers to exhibit their confidence will result in weak understanding of mathematical concepts of the subject there by reducing the teacher the capability of progressing children learning in mathematics” (McCullouch, 2016, p. 9). Mathematics teachers that perceived both statistics and geometry as difficult courses may experience such challenges during teaching which affect both guided discovery instruction and traditional approach of teaching methods in classrooms. It reported that more expert mathematics teachers at the elementary school understand students’ mathematical thinking in solving problems in mathematical concepts and teaching plans than the non-expert teachers. Visualization of Information The quality of geometry instruction has significantly increased over time. When students get older, they frequently stop paying attention to early geometry (Maschietto, Jones, & Doze, 2017, p. 562). According to Sarama, Clements, Wolfe, and Spitler (2016), one of the most efficient ways to assess teachers’ subject-matter expertise and level of comfort in a mathematical environment for effective learning and teaching is through their ongoing professional development.

Challenges Associated with Students Towards Learning of Circle Geometry

The more confident pupils are, the better their attitudes toward learning mathematics will be, and the more probable it is that they will continue studying mathematics at a higher level of education in the future (Van Damme, Van den Noortgate, Kiwanuka, & Reynolds, 2020).

Students are the major people that are considered for both teaching and learning in the society, but they are being influenced by certain variables that affect both guided discovery instruction and traditional method of teaching as being incorporated into the educational environment. In a study regarding Geometrical proofs exhibited by Weber (2017), agreed that students' ability to "construct Geometric figures they had just proven were impossible to construct" (Weber, 2017, Slide 20). Students' thought producing one proof as an example was a proof, they also acclaimed that mathematical proofs must be in two columns and symbols in which that proofs must not contain of written paragraphs and written statement, but not doing self-reasoning of their own "students are utterly perplexed by seemingly arbitrary rules associated with proof" (2017, slide 20). Geometrical or mathematical proofs must be brief and concise with logical meanings of reasoning by students. Studies on literature suggest that, the book of Netz's (1999), accounted for the Shaping of deduction in Greek Mathematics "shows how manuscript diagrams are different from what we see in today's editions" (Saito, 2017, p.49). In the ancient day analyses on text were done which included less writing in mathematical proofs and where instructors thoroughly studied texts of speech and writing to fill in gaps (Saito, 2017). Studies by Weber (2017), accounted that "if proofs are only in place to increase psychological confidence, then students are rational to prefer non-deductive evidence such as Geometer's Sketchpad demonstration" (slide 139). During studies in learning institutions a lot of many students are apprehended with fear and tension as a result of testing situations and difficulties and challenges associated with mathematics as a subject Levine, (Borgonovi, Guerriero, Foley, Herts, & Beilock, (2017). In the research of Mammarella, Caviola, and Dowker (2019), agreed on the major forms of anxiety that involves psychomotor, affective, and cognitive as well as responses of behaviours that causes negative problems in the near future. The negative feelings experienced by students in regards towards tension and apprehension in problem solving mathematics in both academic situations and normal life results in low academic achievement (Mammarella, Caviola, & Dowker, 2019).

Remedies That Can be Employed in Solving Concepts of Circle Theorems by Incorporating or Using Them with Guided Discovery Method on Students' Achievement

Mathematical Games

Teachers incorporating guided discovery method with regard of using mathematical games are the ways employed by teachers to incorporate in teaching concepts in circle theorems for learners understanding and also as a measure to facilitate smooth effective among students within the atmosphere of the school. In the study of Herbst, Gonzalez, and Macke (2005), agreed that with the aim of investigating into preparation of teachers towards how they may use students on the ground to define a

figure in a meaningful way. A study involving 53 students from an SHS Geometry class used a game called "Guess My Quadrilateral" to assess the impact of students' prior knowledge of quadrilateral forms on their performance. The game focused on the neighbourhoods of a special quadrilateral and its differences. The results showed that students were able to evaluate the characteristics of quadrilaterals based on their mathematical games during their studies.

The use of mathematical games for effective teaching and learning in a classroom setting is supported by additional research by Foster and Shah (2015), researcher utilized PCaRD activities in a mixed-methods study at senior high schools, implementing three mathematical games for a year to illustrate effective teaching methods. In the study of Foster and Shah (2015), demonstrated the use of both pre-and post-tests as a measure in determining the achievements of gains in the two groups. Their findings revealed the implementation of the PCaRD model aids students' understanding and learning of geometry with inclusions of circle theorems. According to research by Shah and Foster (2015), putting the PCaRD model into practice enables teachers to use games as resources for their lessons. Geometry may be taught and learned utilizing a wide variety of exercises to support students' knowledge of circle theorem principles. The chosen classroom activity depends on the instructional goals, typically influencing students' learning of Geometry.

Using of Diagrams

The guided discovery approach in geometry teaching utilizes diagrams to illustrate concepts, enhancing communication and understanding of common identities and properties in circle theorems Schwartz and Heiser (2006), The study highlights the importance of diagrams in teaching circle theorems, highlighting the role of visual aids in fostering spatial reasoning and understanding. Board diagrams aid students in understanding relationships, and numerous studies have investigated the impact of a child-centred approach on students' achievement Yerushalmy, Chazan and Gorgon (1990), asserted towards the conduction of the use of Geometric diagrams on how senior high school students' academic performance affects them. The "Geometric Supposers" instructional strategy was modified for high school geometry courses between 1984 and 1988. Researchers identified three barriers to students' comprehension: personalized geometric diagrams, confusion with standard diagrams, and different visualization methods. The study found that students who used "The sup poser" in their coursework understood geometric diagram construction better than those who did not.

Incorporating ICT Tools

GeoGebra, and Geometer sketchpad when combined with guided discovery, can enhance students' mathematical understanding and problem-solving skills when integrated into classroom math teaching, according

to a study (Sariyasa, Ardana, & Murni, 2017). Studies by Yismaw and Gurju (2018), address the challenges and interest needs of students' learning capabilities. This application software empowers students to develop their own knowledge through activity-based approaches, enabling them to tackle global problems. (Bray & Tangney, 2017). Innovative teaching tools, such as manipulative physical machines, personal computers, and intuitive whiteboards, are being utilized to support instruction in mathematics-related subjects (Akçay, 2017). GeoGebra is used as an effective teaching and learning tool to help students understand a variety of mathematics topics, such as geometry and calculus, according to (Zulnaidi, Oktavika, & Hulu, 2018). The software program aims to address difficulties in Geometry education by enhancing student understanding, promoting fun class activities, and fostering positive competition among students, according to studies. (Topuz & Birgin, 2020).

METHODOLOGY

Research Design

They are a group of rules, principles, and instructions that are used in conducting the research this study's goal is to establish the particular research design to be followed or employed to use according to (Cohen, Marrion, & Morrison, 2004). The study used a descriptive survey to assess student's achievement in circle theorem concepts, focusing on challenges, learner-centred activities, and teaching resources, incorporating guided discovery teaching methods for teachers (Appendix D). The experimental and control group of students in the study was primarily assigned to receive questionnaires on the challenges they encountered in the study of circle theorems (Appendix C).

Sample and Sampling Technique

The study involved 164 students from two Senior High Schools in the Municipality, namely school (A) and school (B). The experimental group consisted of 89 students, while the control group consisted of 75 students. Convenience sampling was used to select 32 mathematics teachers from both schools, with 12 females and 20 males. School A and School B were chosen through purposive sampling due to their similar rankings with the Ghana Education Service and available ICT resources. Two classes were randomly selected for the control group and the experimental group to reduce errors due to collaboration and idea sharing. This may be likely to remove the possibilities in connection with the John Henry Effect in studies (Frey, 2018). John Henry Effect is the unfair treatment because of the treatment of reactive behaviours by the control group. The two high schools that were a part of the study included 164 students.

Data Collection Procedure

The study used questionnaires to gather data on challenges faced by students in teaching and learning circle theorems. The instruments targeted both control

and experimental groups and included questions on learner-centred teaching strategies and guided discovery-based instruction. The survey used a four-option Likert scale.

Validity

Validity is defined as being precise and accurate in relation to the results of the study (Mugenda, 2003). "The extent to which an instrument measures what it is intended to measure is referred to as validity," explains (Mohajan, 2017, p. 14). Validity is determined by the precision of research tools, which undergo pre-testing to identify strengths and weaknesses. Content, face and concurrent validity ensure the questionnaire addresses the concept being measured. The study aims to identify issues with current learner centred teaching practices as solutions teachers can use to solve concepts of circle theorems by incorporating it with guided discovery teaching in Wa Municipality schools.

Reliability

Reliability, according to Mugenda (2003), shows how processes in the study are repeated with the exact same outcomes. The students' questionnaires and teachers' Cronbach alpha (α) reliability analysis produced a reliability coefficient of 0.836 and 0.707. According to research by Gay, Mills, and Airasian (2011), test items need to have a reliability coefficient greater than 0.60 in order to accurately measure reliability. The reliability test's objective is to assess the consistency of the research tools. As a result, in order to eliminate or change the items, the research tools were enhanced.

Ethical Considerations

Ethical considerations were sought Prior to collecting data, the Institutional Review Board of C.K. Tedam University of Technology and Applied Sciences was consulted. The Wa Municipal Director of Education was asked for permission. Additionally, permission was sought from the headmasters and mistresses of the two SHS in the Wa municipality.

Data Analysis Procedure and Process

The data had to be analyzed using a variety of methods. Descriptive statistics were used in the data analysis. For the analysis section's responses to research questions 1 and 2, frequency distribution tables were also utilized. RQ 1, the findings were presented in two ways: the teacher-related challenges or difficulties students encounter with circle theorem concepts, and the students' own personal difficulties or challenges during circle theorem learning in Tables 1 and 2. RQ 2, the findings were presented in two ways: the use of learner-centred teaching resources and other appropriate learner-centred teaching methods teachers used to solve concepts of circle theorems by using the guided discovery teaching method as displaced in tables 3 and 4.

RESULTS AND DISCUSSION

RQ 1 was ‘What are the challenges affecting students’ achievement in teaching and learning of circle theorems among Wa Municipality SHS?’. The research question’s main objective was to gather students’ opinions on the difficulties they encountered during the study of circle theorems. Questionnaires were developed for both the experimental and control group’s one-hundred and sixty-four (164) students in order to collect data on the difficulties they encountered while learning of circle theorem concepts see (appendix C). The findings were presented in two ways: the teacher-related challenges or difficulties students encounter with circle theorem concepts in relation to the study of circle theorems and the students’ own personal difficulties or challenges during the study of circle theorems. The researcher also adopted Ofori and Dampson’s (2012), interpretation of a four-point Likert-type scale, where a response of

1.00-1.99 is considered low, a response of 2.00-2.99 is considered moderate, and a response of 3.00-3.99 is considered high.

Teacher-Related Challenges or Difficulties Students Encounter Towards Teaching and Learning Concepts in Circle Theorems

The study involved ten closed-ended questionnaires for students to identify challenges they face when learning circle theorems. The questionnaires were completed using the Likert scale, with 1 indicating strong agreement, 2 indicating agreement, 3 indicating disagreement, and 4 indicating strong disagreement. The questions included understanding the inscribed angled circle theorem, how teacher-dominated methods, explanations, diagrams, and the topic of circles were challenging. The findings are presented in Table 1.

According to the results of the teacher related challenges

Table 1: Teacher-related challenges or difficulties students encounter towards teaching and learning of concepts in circle theorems

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree		Mean	Standard Deviation
	N	%	N	%	N	%	N	%		
Teacher-dominated method makes it difficult for me to understand the inscribed angle theorem.	74	45.1	65	39.6	25	15.2	0	0	1.70	0.72
My classroom teacher finds it difficult to explain concepts on circle theorems to aid my understanding.	67	40.9	70	42.7	12	7.3	15	9.1	1.85	0.91
Proving in circle theorems makes me feel not comfortable to understand Geometry.	78	47.6	65	39.6	8	4.9	13	7.9	1.73	0.88
The ability to see diagrams are not clear for me to establishes the connection between angles in a circle and a tangent.	78	47.6	68	41.5	15	9.1	3	1.8	1.65	0.72
Circle theorem as a topic do not help me to construct angles in a semi-circle.	85	51.8	59	36.0	14	8.5	6	3.7	1.64	0.79
Mean of means									1.71	0.80

Source: Authors Field Work, 2022

students encounter during the study of circle theorems demonstrates that, (139) students representing 84.7% agreed that teacher-dominated method makes them difficult to understand the inscribed angle theorem with respective to the teaching of circle theorems, whereas (25) students indicating 15.2% only disagree with that. The item also recorded a mean and standard deviation (1.70, 0.72) with a low response rate. The students or learners also confirmed that my classroom teacher finds it difficult to explain concepts of circle theorems to aid my understanding with (137) students accounting 83.6% students indicating and of which (27) students showing disagreement representing 16.4% with a low response of a mean and standard deviation (1.85, 0.91). Majority of students also significantly agreed that proofs on circle theorems makes me feel not comfortable to understand Geometry of 143 students representing (87.2%), students

claiming that it is challenged during teaching and studying of circle theorems and 21 students revealing (12.8%) students strongly showing disagreement. The item also denoted a low response with a mean and standard deviation (1.73, 0.88). The findings also agreed that 146 students representing (89.1%), learners affirmed that the ability to see diagrams are not clear for me to establishes the connection between a circle and a tangent, whereas only 18 students representing (10.9%), students showing a form of total disagreement. There was a low response on the item with (M =1.65; SD = 0.72).

Many students also testify that, circle theorem as a topic do not help me to construct angles in a semi-circle of a total number of 144 students representing (87.8%) students whereas 20 students indicating (12.2%), learners showing disagreement. They were a low response on the item with (Mean = 1.64; Standard deviation = 0.79).

The average of means also recorded a low response on the item with ($M = 1.71$; $SD = 0.80$), and this shows low response rate that teacher related challenges or difficulties students encounter towards teaching and studying of concepts in circle theorems have a low effect on their subpar performance. According to the study's findings show that the teacher related challenges pose a low significantly effect in teaching concepts of circle theorems. Studies proposed by McAndrew, Morris, and Fennell (2017), indicate that teaching Euclidean geometry is one of the most challenging parts of the unit. This reflects on Students' difficulties in mathematics are associated with this teacher centred approaches towards concepts in mathematics has been cited by some articles of (Tay & Mensah-Wonkyi, 2018). The findings are also in consistency with the study of Nsengimana, Théophile, Leon Rugema Mugabo, Ozawa Hiroaki, and Pheneas Nkundabakura, (2020), revealed that greatly persistence of teacher- centred method of teaching in both senior and primary schools contribute massively to students' poor performance within the sub-Saharan Africa of mathematics education as well as other topics like Geometry.

The idea teachers find it difficult to explain concepts of circle theorems reflects on the findings of "The inability of mathematics teachers to exhibit their confidence will result in weak understanding of mathematical concepts of the subject there by reducing the teacher the capability of progressing children learning in mathematics" (McCullouch, 2016, p. 9). Mathematics teachers that perceived both statistics and geometry as difficult courses may experience such challenges during teaching which affect both guided discovery teaching and traditional approach of teaching methods in classrooms. This corroborate with the results of the study that Ghanaian educational institutions pedagogical content knowledge adapted towards teaching and learning mathematics including Geometry education is woefully inadequate, which need an urgent attention to curb the situation according to research findings (Wilmot, Yarkwah, & Abreh, 2018). The claim on students indicating that, proofs on circle theorems makes them feel not comfortable to understand Geometry and this reflects on the findings with the level of comprehension in Euclidean geometric proofs and circle theorems concepts are both problems associated with teachers and learners, because it is considered as a difficult and challenging according to studies (Novak & Tassell, 2017). Moreover because of all these challenges and problems associated with circle theorems, many teachers and students are afraid of teaching and studying the topic of circle theorems. This is the reason given as to why a lot of students have trouble with circle or geometry proofs during class (Mwadzaagati, 2017).

The ability to see diagrams are not clear for me to establishes the connection between a circle and a tangent as a teacher-dominated problem reflects research findings on studies done on van Hiele levels on Ghanaians final

year learners before completing senior high school on their thinking abilities on Geometry reveals that failure of learners to progress their level of thinking is likely to be linked to learners' inability to understand vocabulary of Geometrical constructions (Asemani, Asiedu-Addo, & Oppong, 2017). Many students struggle to visualize concepts in Geometrical construction by teachers, because some of them cannot identify properties of Geometrical objects by comparing them based on their relevant previous knowledge. The claim on learners' responses that circle theorem as a topic do not help them to construct angles on a semi-circle which goes to affirmed studies conducted by Fabiyi (2017), research findings revealed the difficulties of Geometrical construction exhibited by students with regards to the following areas of circle theorems for instance congruent triangles, locus construction as well as theorems involving triangles and circles of which reasons was given as: inadequate instructional materials, methodologies teachers' employ in teaching concepts of circle theorems, and attitudes demonstrated by students in learning Geometry. It is agreed upon the findings from the study that teacher related challenges students' encounter towards teaching and learning of circle theorems had a low effect on students' subpar performance of studying circle theorems.

Students Own Personal Problems They Encounter Towards Teaching and Learning of Circle Theorems

The study investigates students' personal challenges with circle theorems, including lack of mathematical instruments, confidence in problem-solving, difficulty in understanding the topic, and fear of tangent theorems. Five questionnaire items reveal these issues, with Table 2 illustrating these challenges.

According to the study's findings, students face unique difficulties when learning and teaching circle theorems. The findings from the results that 139 students accounting (84.7%) students agreed that I do not have mathematical instruments to construct diagrams on circle theorems and 25 students indicated (15.2%), students showing disagreement with a low response of ($M = 1.69$; $SD = 0.72$), goes to affirmed studies conducted by Fabiyi (2017), research findings revealed the difficulties of Geometrical construction exhibited by students with regards to the following areas of circle theorems, for instance, congruent triangles, locus construction as well as theorems involving triangles and circles of which reasons was given as inadequate instructional materials, methodologies teachers' employ in teaching concepts of circle theorems, and attitudes demonstrated by students in learning Geometry. There were many students also claiming that I lack confidence in my capacity to use circle theorems to solve problems as a student with 136 students showing (83%), students agreeing to the findings and 28 students representing (17%), students not agree to the statement recording a low response on the item with ($M = 1.85$; $SD = 0.92$) and this finding is agreed upon studies on Weber (2017), accounted for "if proofs

Table 2: Students own personal problems they encounter towards teaching and learning of circle theorems

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree		Mean	Standard Deviation
	N	%	N	%	N	%	N	%		
I do not have mathematical instruments to construct diagrams on circle theorems.	76	46.3	63	38.4	25	15.2	0	0	1.69	0.72
I lack confidence in my capacity to use circle theorems to solve problems as a student.	67	40.9	69	42.1	13	7.9	15	9.1	1.85	0.92
I perceived to find circle theorem as a difficult topic.	85	51.8	59	36.0	14	8.5	6	3.7	1.64	0.79
I do not revise or practice questions on circle theorems that are constructed by my classroom teacher.	78	47.6	68	41.5	15	9.1	3	1.8	1.65	0.72
Fear, tension, and apprehension makes me difficult to understand the tangent theorem.	93	56.7	69	42.1	2	1.2	0	0	1.45	0.52
Mean of means									1.66	0.73

Source: Authors Field Work, 2022

are only in place to increase psychological confidence, then students' are rational to prefer non-deductive evidence such as Geometer's Sketchpad demonstration" (slide 139). In a study regarding Geometrical proofs exhibited by Weber (2017), agreed that students' ability to "construct Geometric figures they had just proven were impossible to construct" (Weber, 2017, Slide 20).

144 students representing (87.8%), students also personally agree that I perceived to find circle theorem as a difficult topic and 20 students accounting (12.2%), students did not agree from the results of the research indicating a low response with ($M = 1.64$; $SD = 0.79$), affirming to the study's findings confirmed that majority of students' get discouraged in studying Geometry due to lack of understanding of it as a topic in the educational institutions in the case of Idris as referenced by (Utami, Mardiyana, & Pramudya, 2017). The findings on the study validated that concepts taught by instructors of the following concepts in Geometry are perceived as difficult and challenging to students' including coordinate geometry, latitude and longitude, solid figure volume and surface areas, congruent triangles, locus construction as well as theorems involving triangles and circle theorems according to studies posited by the following researchers (Egwu, Asuque, & Ofori, 2018). The study's findings show that 146 students representing (89.1%), students also agree with the statement that, I do not revise or practice questions on circle theorems that are constructed by my classroom teacher whereas 18 students indicating (10.9%), students did not agree with the statement with a low response rate of ($M = 1.65$; $SD = 0.72$). This support the findings on studies conducted on the results of WASSCE reveals that teachers are at the advantaged of completing their topics in the teaching syllabus for finishing's sake, and not teaching concepts in circle theorems or geometry well which is leading to students' low achievement in circle theorems (Chisenga & Mulenga, 2019). This contributes negatively in affecting them not

been able to revise or practice questions constructed by their classroom instructors, which later affects their performance in circle theorems.

The majority of students also confirmed that fear, tension, and apprehension make it difficult to understand the tangent theorem with 162 students representing (98.8%), learners and only 2 students accounting (1.2%), students not showing agreement with a low response of the ($Mean = 1.45$; $Standard deviation = 0.52$). The average means was also low in term of the response of the respondents ($M = 1.66$; $SD = 0.73$) revealing that students' own personal problems they encounter with teaching and learning circle theorems is having a low effect on their performance. This statement affirmed the findings of the study that negative feelings experienced by students in regards towards tension and apprehension in problem solving mathematics in both academic situations and normal life results in low academic achievement (Mammarella, Caviola, & Dowker, 2019). The findings also confirmed that during studies in the learning institutions a lot of many students are apprehended with fear and tension as a result of testing situations and difficulties challenges associated with mathematics as a subject (Foley, Herts, Borgonovi, Guerriero, Levine, & Beilock, 2017). The findings also support the study of Mammarella, Caviola, and Dowker (2019), agreed on the major forms of anxiety that involves psychomotor, affective, and cognitive as well as responses of behaviours that causes negative problems in the near future. There was a low response rate according to the study's findings, the majority of students face unique issues with their attitudes, concerns, and feelings regarding the learning of circle theorems had a low effect on their performance.

RQ2 was 'What remedies can be employed in solving concepts of circle theorems using guided discovery teaching method on students' achievement among SHS's in Wa Municipality? The primary intent of this research project was to gather opinions on experts in mathematics

education as well as particularly, mathematics teachers' opinions on the type of other learner centred teaching activities that can be taught by using guided discovery teaching strategies employed to solve concepts of circle theorems to aid students' conceptual understanding. Questionnaires were developed for thirty-two mathematics teachers of which 17 are in school (A) as well as 15 been in school (B) see (Appendix D) for the questionnaires formulated for teachers were designed to seek their opinions regarding other learner centred activities teaching strategies that can be taught by incorporating it with guided discovery method to solve concepts of circle theorems. The findings were presented in two ways: the use of learner centred teaching resources or teaching aids and other appropriate or suitable learner centred activities that can be taught by incorporating it with guided discovery method. The researcher also used Ofori and Dampson's (2012) interpretation of a four-point Likert-type scale, where a response of 1.00-1.99

is considered low, a response of 2.00-2.99 is considered moderate, and a response of 3.00-3.99 is considered high.

The Use of Learner-Centered Teaching Resources or Teaching Aids

The study analysed teachers' use of learner-centred activities and teaching strategies to solve circle theorem concepts using a guided discovery teaching method. Ten closed-ended questionnaires were developed for two schools, focusing on the effectiveness of various teaching aids. The questionnaires asked teachers about their use of diagrams, cut-out shapes, mathematical instruments, and other ICT tools in teaching circle theorems. The results of these surveys are presented in Table 3. The study aims to provide insights into how teachers can effectively teach circle theorem concepts in the classroom.

According to the results of the learner centred teaching resources teachers incorporate as remedies to teach concepts of circle theorems by incorporating it with guided

Table 3: The use of learner-centered teaching resources as remedies teachers use to solve concepts of circle theorems by using guided discovery teaching method

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree		Mean	Standard Deviation
	N	%	N	%	N	%	N	%		
Illustration of diagrams on the marker board aid visualizations which helped me teach circle theorems effectively.	13	40.6	16	50.0	2	6.3	1	3.1	1.72	0.73
Forming cut-out shapes of plane and solid figures on cardboard helped me teach the inscribed angle theorem.	13	40.6	14	43.8	4	12.5	1	3.1	1.78	0.79
Mathematical instruments helped me to construct angles in a circle.	12	37.5	15	46.9	3	9.4	2	6.3	1.84	0.85
Incorporating the use of other ICT tools on Geometry assist me to draw the various positions of a right-angled triangle on circle theorems.	15	46.9	13	40.6	2	6.3	2	6.3	1.72	0.85
Mean of means									1.77	0.81

Source: Authors Field Work, 2022

discovery teaching method demonstrates that, 29 teachers representing (90.6%), teachers agreed that illustration of diagrams on the marker board aid visualizations which helped me teach circle theorems effectively and 3 of the teachers accounting (9.4%), teachers also disagree with the statement. The item also recorded a low response (M = 1.72; SD = 0.73). Majority of teachers also agree that forming cut-out shapes of plane and solid figures on cardboard helped me teach the inscribed angle theorem with 27 teachers representing (84.4%), agreeing to the statement and 5 teachers indicating (15.6%), teachers showing disagreement. The item also recorded a low response rate (M = 1.78; SD = 0.79).

The teachers also confirmed that, mathematical instruments helped me to construct angles in a circle with 27 teachers accounting (84.4%), teachers also agreeing with that whist 5 teachers showing (15.7%), teachers

showing no disagreement with the statement, there was also a low response rate of the item with (M = 1.84; SD = 0.85). A quite number of 28 teachers representing (87.5%), teachers also testify that incorporating the use of other ICT tools on Geometry assist me to draw the various positions of a right-angled triangle on circle theorems with 4 teachers indicating (12.6%), teachers also did not agree with the statement, there was also a low response rate of the item with (Mean = 1.72; Standard deviation = 0.85). The average of means also recorded a low response on the item with (M = 1.77; SD = 0.81) indicating that, the use of learner centred teaching resources or teaching aids as remedies teachers use to solve concepts of circle theorems by using guided discovery teaching method indicated a low response rate. According to the study's findings, a majority of teachers concur that using learner centred teaching aids

or instructional materials as remedies or strategies helps teachers solve concepts circle theorems by using guided discovery teaching method.

The study's conclusions are consistent with research asserted by Yerushalmy, Gorgon and Chazan, (1990) that looked at the impact of utilizing geometric representations on students' performance in senior high school. They also are consistent with the claim that teachers used diagrams drawn on a whiteboard to successfully teach circle theorems as guided discovery teaching remedy or strategy. The study's results also corroborate a study by Schwartz and Heiser (2006), who found that students' spatial reasoning skills are aided in their learning of circle theorem concepts by diagrams that illustrate key ideas. The findings also corroborate research that was modified to teach high school geometry courses between 1984 and 1988 under the influence of the "Geometric Supposers," an inquiry- or activity-based instructional strategy. After identifying three crucial barriers to students' comprehension in the classroom when they were looking at and interpreting diagrams, the researchers used "Geometric Supposers" in their lesson plans. Using geometric diagrams customized for each student, the use of frequent diagrams that lead to a lot of confusion with standard diagrams, and the visualization of one particular diagram in various ways all contributed to the three main factors that were discovered.

The results concur with research by Donevska-Todorova (2015), who claimed that using a variety of diagrammatic representations allowed students' comprehension of a particular mathematical concept. The results also supported research on the discovery approach, in which students use cut-out shapes and the Cartesian plane to manipulate geometric figures practically and identify their properties in order to create images from pre-images (Serin, 2018). Some of these studies on circle geometry and the idea of teachers creating angles in a circle using mathematical tools have focused on the creation of geometrical objects and shapes as well as the application of geometrical problems to the improvement of learners' reasoning.

Additionally, the results support a study by Canada and Blair (2006), that looked at, as an illustration of one of these, the points where a circle and a square meet, two circles a square can intersect anywhere. The study involved both students and pre-service teachers. The study's tasks were created by the authors to assist students in developing mathematical arguments, theories, and relationships between mathematical concepts.

Building a six-point intersection and locating additional potential intersection points were part of the study's assignment. Researchers for instance Canada and Blair (2006), found that even when circles and squares were not drawn precisely, students could still recognize potential 0, 1, 2, and 3 points of intersection. Canada and Blair (2006), also observed that depending on how accurately students drew 5-, 6-, and 7-point intersections, the conclusions reached varied. The researchers came to the

conclusion that students could make precise discoveries and deductions about geometric concepts when diagrams were drawn precisely.

According to teachers, drawing right-angled triangles in various positions using circle theorems is made easier by integrating other ICT tools into geometry lessons. When used as a practical ICT tool for teaching and learning, GeoGebra has been found to improve students' understanding of a variety of mathematics topics, including geometry and calculus, according to research by (Zulnaldi, Oktavika, & Hulu, 2018). Modern teaching and learning tools include mathematical tablets, physical machines for manipulating objects, laptops, intuitive whiteboards, and many others that support instruction in mathematics topics (Akca, 2017).

The findings support Ertekin's (2014), study on the impact of Cabri 3D on pupils' geometrical skills. In this study, objectives, which were to ascertain whether students could correctly formulate an equation for a particular plane and sketch its graph, are also supported by the study's findings. The experimental group and the control group were the two groups. In the experimental group, but not in the control group, Cabri 3D was used to instruct the students. A study found that students who received instruction using the Cabri 3D had better success identifying special planes and their corresponding normal vectors. Learners who were instructed using Cabri 3D also did a better job of creating the corresponding diagrams for the special planes. According to the study's findings, teachers can help students' conceptual understandings by using instructional materials or teaching aids as strategies when teaching circle theorem concepts using guided discovery teaching method

Other Suitable or Appropriate Learner-Centred Teaching Strategies That Can be Taught by Using or Incorporating Them with Guided Discovery Methods for Solving Concepts of Circle Theorems to Aid Students Understanding

The study explores learner-centered teaching strategies for teaching circle theorems using the guided discovery teaching method. It identifies six strategies: providing exercise, group activities, mathematical games, role play, guided learning, and brainstorming. The questionnaire items highlight the effectiveness of these strategies in teaching circle theorems, establishing connections between tangents and circles, and diagnosing problems using guided learning.

The study's findings reveal that other suitable or appropriate learner-centred activities that can be taught by teachers employing the guided discovery teaching method in solving concepts of circle theorems demonstrated that, 28 teachers (87.5%), teachers agree on the idea of providing enough exercise on circle theorems with activities involved helped me to teach circle theorems with 4 teachers revealing (12.5%), teachers showing disagreement. Additionally, the item had a low response rate of (Mean = 1.69; Standard deviation = 0.78). There

Table 4: Other suitable or appropriate learner-centered teaching strategies that can be taught by incorporating them with guided discovery methods for solving concepts of circle theorems to aid students' understanding

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree		Mean	Standard Deviation
	N	%	N	%	N	%	N	%		
Providing enough exercises on circle theorems with activities involved helped me to teach circle theorems.	15	46.9	13	40.6	3	9.4	1	3.1	1.69	0.78
Putting students in groups and assisting them to perform activities on circle theorems helped me establish the connections between an angle in a tangent and a circle.	13	40.6	14	43.8	3	9.4	2	6.3	1.81	0.86
The use of mathematical games motivates me to teach concepts of circle theorems.	19	59.4	8	25.0	4	12.5	1	3.1	1.59	0.84
The use of role play guide me to teach circle theorems effectively.	16	50.0	15	46.9	1	3.1	0	0	1.53	0.57
Guided learning helps me to diagnose the problems on circle theorem.	18	56.3	12	37.5	2	6.3	0	0	1.50	0.62
Brainstorming on concepts of circle theorems assist me to teach Euclidean Geometry.	12	37.5	13	40.6	4	12.5	3	9.4	1.94	0.95
Mean of means									1.68	0.77

Source: Authors Field Work, 2022

is also affirmation from 27 teachers showing (84.4%), teachers agreeing to the statement that, putting students in groups and assisting them to perform activities on circle theorems helped me establish the connections between angles in a tangent and a circle but only 5 teachers revealing (15.7%), teachers did not agree, with the item being indicating a low response rate of ($M = 1.81$; $SD = 0.86$). Twenty-seven (27) representing 84.4% of teachers also agree that the use of mathematical games motivates me to teach concepts of circle theorems and 5 teachers indicating (15.7%) teachers disagreed with the assertion. Additionally, the item had a low response rate of the item of which it is recording ($M = 1.59$; $SD = 0.84$).

The vast majority of teachers also claim from the findings of the results that the use of role plays guides me to teach circle theorems effectively with 31 teachers representing (96.9%), showing agreement and 1 teacher indicating (3.1%) showing no agreement. There was also a low response rate of ($M = 1.53$; $SD = 0.57$). Confirming the results of the study, it is agreed by 30 teachers (93.8%), of teachers that guided learning helped me to diagnose the problems on the circle theorem, and 2 teachers (6.3%), teachers did not agree with the statement. There is an indication of a low response rate ($M = 1.50$; $SD = 0.62$). It is affirmed that brainstorming on concepts of circle theorems assisted me in teaching Euclidean Geometry with 25 teachers representing (78.1%), teachers showing agreement and 7 teachers indicating (21.9%), teachers did not agree to that. There was a low response rate among the respondents with regards to the standard deviation and mean with ($M = 1.94$; $SD = 0.95$). The mean of

means also recorded a low response rate ($M = 1.68$; $SD = 0.77$), on other suitable or appropriate learner-centred activities that can be taught by employing the guided discovery teaching method by teachers as a teaching method for solving circle theorems concepts.

Based on the study's results, it supports the studies with regards to teachers claiming on providing enough exercise with activities involved help to aid students understanding on the more confident pupils are, the better their attitudes toward learning mathematics will be, and the more probable it is that they will continue studying mathematics at a higher level of education in the future (Reynolds, Kiwanuka, Van Damme, & Van den Noortgate, 2020). The results back up teachers' claims that grouping students and helping them work through circle theorem activities helped teachers clarify the connection between angles in a circle and a tangent. In addition, studies have confirmed the importance of a child cooperating with friends in influencing cognitive development. Individuals' cognitive development is nourished when they collaborate with others. In other words, studies of cognitive development show a progression from behaviour's organized by others to self-regulated behaviour's (Senemoglu, 2012).

The results back up Slavin's (2010), research, which showed that cooperative learning increases motivation and improves student performance. Both assert the core of cooperative learning is peer interaction and collaboration. They value the in-depth justifications that come from cooperative learning. The idea of teachers using mathematical games as an activity-based learning

strategy motivates them to teach effectively reflects on the findings with regards to the study of Herbst, Gonzalez, and Macke (2005), who agreed that the aim of investigating the preparation of teachers may use students in the ground to define a figure in a meaningful way. Students took part in a game of geometric mathematics called “Guess My Quadrilateral.” The game’s objective was to assess the impact that the student’s prior knowledge of quadrilateral forms had on their performance. The findings also support other studies conducted by Foster and Shah (2015), who agree on how mathematical games could be employed in both the teaching and learning school atmosphere. Play, educational activities, reflection, and discussion (PCaRD) were used by the researchers as models for strategic teaching. According to research results presented by Mintah (2016), in the classroom, particular techniques are employed to teach mathematics. Effectively even though they are viewed as activity-based strategies that anyone can use. These methods include role-playing, brainstorming, guided learning, think-pair, discovery or inquiry method, project method, and inductive-deductive method.

Major Findings

1. Teacher-related challenges that students encounter as well as the students’ own individual difficulties in comprehending circle theorem concepts have a low effect rate on their subpar performance and understanding recording a mean of 1.71 on the part of teacher-related challenges and 1.66 on the part of students own personal problem. Despite the fact that most students agreed with the ten items on the test, a sizeable portion of students disagreed. As a result, the performance range or rate was low.

2. Teachers implementing a variety of other suitable or appropriate learner-centered teaching techniques as well as learner-centered teaching resources or teaching aids with the effort of incorporating them with guided discovery teaching method to introduce the ideas of circle theorems to students, will enhance their capacity for conceptual understanding and problem-solving recorded a low effect rate on their understanding recording a mean of 1.77 on learner-centered teaching resources and 1.68 on other suitable learners centered method, even though the majority of the teachers agreed with the ten test items, a sizable portion of them disagreed.

CONCLUSIONS

The study’s findings led to the following three deductions.

1. Firstly, it can be deduced that there is a low effect rate on academic achievement in concepts of circle theorems caused by both the teacher-related challenges and students’ own individual problems with circle theorems.

2. Secondly, there is also a low effect rate with the aid of teachers implementing a variety of appropriate or suitable learner-centered teaching techniques as well as learner-centered teaching resources or teaching aids when they are incorporated with the guided discovery

method to improve students understanding of geometric concepts and problem-solving abilities.

3. Thirdly, the study’s findings show that teachers implementing modern teaching approaches, like brainstorming, role play, mathematical games, illustration of diagrams on cardboard, as well as putting students in groups and incorporating them with guided discovery learning into teaching and learning, have been shown to be more effective with a low rate. As a result, these modern methods can be used in classroom instruction as blended instruction. Prior studies have also given favor to the use of guided discovery teaching strategies in mathematics education. Therefore, it can be said that students benefit from a geometry curriculum that emphasizes activities, especially when teaching Circle Theorems.

RECOMMENDATIONS

1. Using different learner centred teaching strategies of circle geometry must be incorporated with guided discovery teaching methods as a dynamic educational teaching method to be implemented by curriculum developers into the mathematics senior high school curriculum. This will help teachers support students in understanding geometric concepts and gain proficiency in math problem-solving and difficulties associated with circle geometry. This will have a significant impact on the students’ performance. Teaching and learning activities could include guided discovery teaching method which increases Students’ interest in mathematics will increase as a result of this. This could be accomplished by giving students access to resources like computers, math tools, and educational applet devices.

2. To enhance the performance of SHS students, it will be most advantageous to introduce learner centered teaching strategies onto guided discovery teaching methods to pre- and in-service teachers, through promotion by Ghana Education Service and/or other stakeholders in the education sector, this introduction may be distributed through workshops and seminars. Math teachers’ instructional techniques and skills will improve as a result.

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