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Comparative Assessment of Comminution Energy Equations (Models) Using Some Selected Legumes, Tubers, Cereals and Sea Food

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ABSTRACT

The quest to predict more precisely energy that is just enough to achieve closely any desired final particle size of food material subjected to comminution is necessary. This is to avoid energy wastage, obtain finer product size(s) that are uniform and can encourage easy mixing, dehydration/drying, etc. In this regard, some selected legumes (soybeans and beans), cereals (sorghum, millet, corn and wheat), sea food (crayfish) and tubers (cassava and yam) were subjected to comminution through the application of some major energy equations (models); as they find various applications in food industries. Four energy equations (models) namely the Kick's, Rittinger's and Bond's for size reduction and the OruaAntia's minimum energy equation for mass-size reduction were employed. The constant in each equation (model) to be applied in grinding of selected food materials was determined and used in obtaining the required specific energy of comminution needed to accomplish desired final product size(s). The corresponding grinding time expected to achieve the desired final product were computed and used in operating the grinding machine. Results revealed that the OruaAntia's minimum energy equation for mass-size reduction operation may be applied on the selected food materials to achieve very closely any desired final average particle diameter with a percentage deviation of 1.66% followed with Rittinger's equation having average percentage deviation of 6.88%. Technical analysis re-affirm that Kick's equation could only achieve coarse particles as the final particle size showed average percentage deviation of 38.40%, while Bond's equation may be limited to prediction of coarse and intermediate particles; since the final particle size average percentage deviation was 16.19%. Besides using Rittinger's energy equation to obtain fine particles, the use of Orua Antia minimum energy equation may possibly further achieve desired finer particle size(s).

INTRODUCTION

The amount of energy used for grinding materials may depend on the materials physical characteristics, machine geometrical and kinematic parameters used for the grinding. Size reduction operation is very important in food sector as it enhances powder mixing, reduction in sedimentation rate among several things (Sushaut & Archana, 2013; Song., Mycong., Hyong., & Won., 2014; Dabbour., Bahnasawy., Ali., & El-Haddad., 2015; Mulla., Hajare., & Doijad., 2016). During size reduction, particles may experience elastic and inelastic deformation. Hence, requires appropriate energy to produce product that may be averagely uniform in size or categorized into distinct size ranges. Thus, accurate estimation of energy requirements necessary for size reduction equipment is economically essential in many aspect (Antia., Obahiagbon., Aluyor., & Ebunilo., 2013; Antia & Aluyor, 2018).

In an effort to minimize cost coupled with achieving the desire size range of material, various equations (models) have been proposed and developed based on the differential energy (dE) required to produce a small change (dX) in size of a unit of a material x (Akinoso., Lawal., & Aremu., 2013; Ndukwu., Nwakuba., & Henry., 2016; Antia, 2021, pp. 672-673) as:

$$dE/dX \propto x^{(-n)} \quad (1)$$

$$dE= -kx^{(-n)}\, dX \quad (2)$$

Where, k and n are constant. n=1,2,3⁄2for kicks, Rittinger's and Bond's concept respectively.

These expressions are as follows:

$$E_k=K_k\,[\ln x_1/x_2\,] \quad (3)$$

$$E_R=K_R\,[1/x_2\,-1/x_1\,] \quad (4)$$

$$E_B=K_B\,[1/\sqrt{(x_2\,)}-1/\sqrt{(x_1\,)}] \quad (5)$$

Where,E_k,E_R and E_B are energy by Kick's,Rittinger's and Bond's respectively and may be expressed in J or J/kg or kWh/kg or Ws/kg.

K_k, K_R and K_B may be expressed in J or J/kg or kWh/kg,Jm or Jm/kg or (kWh.m)/kg,$Jm^{(1/2)}$ or $Jm^{(1/2)}/kg$ or $kWh.m^{(1/2)}/(kg)$ respectively as Kick's, Rittinger'sand Bond's constants while x_1 and x_2 are respectively initial and final particle sizes(diameter(s)) in m.

A new dimension considered by Antia., Obahiagbon., Aluyor., & Ebunilo., 2014 in this regard was that energy (dE) is required to produce a small change (dA) in area of a unit material. This approach is because it is easier to measure effectively any material by mass (m) irrespective of it size and shape. Moreso, mass could be related to area (A) as:

$$m=\varrho V=\varrho AT \quad (6)$$

Where, V= volume of the material

ϱ= density of the material

T= thickness of the material

$$\text{Where, } A= m/\varrho T \quad (7)$$

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Hence, the concept was expressed as:

$dE/dA \propto A^{(-n)}$ (8)

$dE= -KA^{(-n)}\, dA$ (9)

$E_{min}= Bm^{(1-n)}/(1-n)$ (10)

Where, n= power index

E_min=Minimum energy required for mass reduction of the materialThe power index n was experimentally determined as n =1⁄2. The minimum energy for the mass reduction was obtained as:

$E_{min}=2Bm^{(1/2)}$ (11)

Where,B = mass index, m= mass of the material

A combination of analytical and empirical approaches to relate size(diameter) of the material resulted in one of the expressions (models) for mass-size reduction operations (Antia, 2020) as:

$E_A=K_{(A3)}\ [1/(D_2^{(3/2)})-1/(D_1^{(3/2)})]$ (12)

Where, $K_{(A3)}$ = Orua Antia Energy constant expressed in $Jm^{(3/2)}$ or $Jm^{(3/2)}/kg$ or KWh. $m^{(3/2)}/kg$

E_A = Orua Antia minimum energy for mass-size reduction of material and may be expressed in J/kg.kWh/kg or Ws/kg

D_1= Initial diameter of the material in m.

D_2 = Final diameter of the material in m.

$K_{(A3)}= (2B\varrho_m^{(-1/2)})/(C_f\, M_f)\ (0.2304)= (0.4608\, B\varrho^{(-1/2)})/(C_f\, M_f)$ (13)

Where, C_f= crushing efficiency

M_f= Mechanical efficiency

Besides the mentioned energy equations (models), various researchers have proposed other models during studies on energy consumption for grinding materials. Mani., Tabil., & Sokhansanj (2004) worked on energy consumption of switch grass, corn stover and wheat straw using hammer mill with screen size of 0.8, 1.6 and 3.2mm. The data obtained fitted a straight line and a second order polynomial respectively at 8 and 12% moisture content of the material as:

$E = k_1+ k_2\, d$ (14)

$E = k_1+ k_2\, d+k_2\, d^2$ (15)

Where, E = specific energy,(kWh⁄t)

d is the hammer mill screen size (mm)

k_1, k_2= constants.

Bitras., Womac., Chevanan., Miu., Igathinathane., Sokhansanj *et al.* (2009) also worked on switch grass, corn stover and wheat straw having respectively the initial mean sizes of 8.3, 7.1, and 8.3 mm. The feeding rate of 41.7 g/s and hammer mill with 3.2 mm screen were used. The data obtained fitted equation given as:

$E_e= K\Delta X_g$ (16)

Where, K is a function of the rotational speed, N [i.e. K=f(N)]

E_e= specific energy (MJ/Mg)

ΔX_g= unit size reduction (mm)

Adapa., Tabilo., & Schoenau (2011) obtained barley straw, canola straw, oat straw, and wheat straw for grinding using two hammer mills. The first had 30 mm screen while the second had 6.4, 3.2, and 1.6 mm screen. The energy equation proposed was given as:

$E=k_1\, S^{(-k2)}$ (17)

Where, E = specific energy, (kWh⁄t)

s= hammer mill screen size (mm)

k_1, k_2= constants.

Miao., Grift., Hansen., & Ting (2011) used a knife mill with screens that ranges from 10mm to 1mm to grind some biomass at two moisture content levels of 15 and 7%. The energy consumption and particle size were related as:

$E=ax^b$ (18)

Where, E= specific comminution energy, J/gx= comminution ratio(ratio of average size of initial biomass to average size of comminuted particles)a and b = regression constants.

In another development, the Kick's, Rittinger's and Bond's equations were used to grind hard wood and soft wood (Naimi., Sokhansanj., Bi., Lim., Womac., Lau *et al.*, 2013). It was observed that the predicted specific comminution energy fitted best using Rittinger's equation followed by Bond's and Kick's equations. Ladan., Flavien., Xiaotao., Jim., & Shahab (2016) used rotary knife mill and Lingocellulose biomass conditioned at 11.5% moisture content coupled with Kick's, Rittinger's and Bond's equations. The experimental data generated fitted best the Rittinger's followed by Bond's and Kick's energy equations for size reduction operations.

In this study, the energy Equations (models) 3,4,5 and 12 were used to assess the most applicable energy of comminution for some selected cereals (corn, millet, wheat and sorghum), legumes (soya beans and beans), tubers (yam and cassava) and sea food (cray fish).

MATERIALS AND METHODS

Equipment, Sourcing and Preparation of Materials

The equipment used includes: vernier calliper, sieves, digital weighing balance, polytene bags, sieve shaker, stop watch and grinder (Hammer-Attrition mill). The selected materials used were cereals (corn, millet, wheat and sorghum), legumes (soybeans and beans), tuber (cassava and yam) and sea food (cray fish,) obtained from Uyo local market, Akwa Ibom State, Nigeria. Each material was cleaned manually to remove any unwanted material on it; and then air dried at 105℃ till constant weight (bone dried mass) was achieved (Antia., Oboh., & Olosunde., 2019). Thereafter, each material was stored for use in this study.

Evaluation of Energy Equations (models) Constants

The grinding machine energy was first determined by reading the power of electric motor used from the machine description panel. A constant time for grinding was chosen to evaluate all the constants in the energy equations (Kick's,Rittinger's, Bond's and OruaAntia's). The energy of the grinding machine was given (MohdRozalli., Chin., & Yusof., 2015; Antia, 2021, pp. 681-692) as:

Energy (J)= power(J⁄s)x time(s) (19)

Therefore, the specific energy E_1^* of the grinding machine powered by the electric motor may be given as:

E_1^*=([Energy (J)])/(mass of material @subjected to grinding, kg)=(Electric motor power (J/s)×@grinding Time (s))/(mass of material @subjected to grinding, kg) (20)

The specific energy E_1^* may be expressed in J/kg or kWh/kg or Ws/kg.

The grinding time chosen and the electric motor power used were 120sec and 0.746kW respectively. The mass (500g) of each selected material to be used per experimental run was chosen and its average initial (feed) diameter was determined. The average final (product) diameter of each selected material after being subjected to grinding was determined using sieve analysis (Antia, 2021, pp. 700-710; Okoro, 2001, pp. 31-53). The energy constants of Equations 3,4,5 and 12 were determined using specific Energy E_1^*, the sample average initial feed diameter of the feed and the average final product diameter.

Estimation of Specific Energy of Comminution and Grinding Time Applicable to Each Energy Equation

In assessment of the energy Equations 3,4,5 and 12 for size reduction operations;three desired final particle sizes were chosen as 300μm, 400μm, and 500μm. Sieve aperture size that correspond to the chosen (desired) final product was used to sieve out each of the selected materials after grinding.The specific comminution energy E_2^* based on the four energy Equations 3,4,5 and 12 were computed for each of the selected samples by using each corresponding energy equation constant obtain in section 2(ii)coupled with the initial average diameter of the feed and each desired (chosen) average diameter of the final product.Each time expected to grind the particles to achieve each of the three (3) chosen sizes (diameters) 300μm, 400μm, and 500μm were calculated based on the Equation 21(MohdRozalli., Chin., & Yusof., 2015, Antia, 2021, pp. 681-692) as:

$$t_g=(E_2^* \times m_s)/P_m \quad (21)$$

Where, E_2^*= Expected specific comminution energy required in (kWh.Kg$^{(-1)}$)

m_s=Chosen quantity of sample to be ground (Kg)

P_m=power of the grinding machine (kW)

t_g=Expected time required for grinding (hr)

The estimated time required to achieve each final product size chosen per selected material was used to run and grind the sample(s) accordingly. The final product of eachselected sample per grinding time was subjected to sieve analysis; in order to determine the most appropriate energy equation for application, to achieve the expected desired size (diameter) (d_2^*) of the final product.

Determination of Particle Diameter for Comparative Assessment of the Energy Equations

Sieving tests were conducted to determine the initial and final average/mean particle size (d_p), of each selected material subjected to comminution. Each test was conducted by placing a series of sieves with progressively smaller mesh sizes on top of each other and passing the sample through the stacked sieve "tower" using a sieve shaker for a given time.The following expressions were used for the analysis (Antia, 2021, pp. 700-710; Okoro, 2001, pp. 54-69):

$$R_f = z/m \quad (22)$$

Where, z=weight of material on a given sieve

m= mass of sample

$$d_{pi}= S_i R_f \quad (23)$$

$$d_p = \sum d_{pi} \quad (24)$$

Where, i=1,2,..... n is the serial number of sieve aperturesincluding the final bottom material n sieved out

S_i=aperture size of the sieve used

d_{pi}=average particle size (diameter) on i sieve

The values of the average/mean diameters obtained using each energy equation on each selected material were compared; to determine the equation that could be used to predict closely the comminution of each selected material. This may be achieved by computing the percentage deviation of actual final average diameter (d_2^*) with predicted (chosen) final average diameter (d_2) of the selected materials subjected to comminution.The equations may be given as:

$$\%\ \text{deviation}\ (E_p)=[(\sum d_2^* - d_2)/N_T]\times 100 \quad (25)$$

Where, N_T=d_2 n

d_2= desired (chosen) final average particle size (diameter), m

n = number(s) of selected material(s) used in testing the energy equations

d_2^* = actual final average particle size (diameter) following comminution,m

$$\text{average}\ \%\ \text{deviation}\ (E_{pa})=(\sum E_p)/N_{TS} \quad (26)$$

Where,N_{TS}=the total number of size(s)(diameter(s)) chosen and used in testing the energy equations(s).

RESULTS AND DISCUSSION

Each constant required as applicable to Kick's, Rittinger's, Bond's and OruaAntia's energy Equations 3, 4, 5 and 12 respectively for each of the selected sea food (crayfish), cereals (sorghum. millet, corn and wheat), legumes (soybeans and beans) and tubers (cassava and yam) wasobtained and are presented in Table 1.

From Table 1, it is observed that at the chosen grinding time 120sec (2min), the grinding machine with power of 0.746kWcoincidently produce final average product size of 600μm for each 500g of the selected materials. This may imply that at commencement of grinding, each equation accommodate this size within two (2) minutes range of grinding time; and beyond which each energy equation may likely start to exhibit its peculiar capability. In assessing each of the energy equations, the specific energy of comminution E_2^* applicable to each of the energy equation considered was calculated based on the initial average feed diameter(s) and chosen final average product size(s) (300μm, 400μm, and 500μm) coupled with their respective determined constant in Table 1. The obtained values of E_2^* using Equations 3, 4, 5 and 12 are presented in Tables 2 to 4.

From Tables 2 to 4, each grinding time per selected material was obtained from Equation 21. Moreso, the desired (chosen) average particle size(s) d_2 and actual average particle size(s) d_2^*obtained after subjecting the

Table 1: Energy equations constants obtained for selected materials with respect to grinding machine power (0.746kW) and time (120sec) used.

	Selected materials (samples)	(d_1) Initial feed diameter (m)	(d_2) Final product diameter (m)	(E_1^*)Specific energy requirement (J/kg or kWh/kg)	Energy equation constants			
					Kick's, K_k (J/kg) ($10^{(-2)}$)	Rittinger's, K_R (Jm/kg) ($10^{(-5)}$)	Bond's, K_B ($Jm^{(1/2)}$/kg) ($10^{(-3)}$)	OruaAntia's, K_k ($Jm^{(3/2)}$/kg) ($10^{(-7)}$)
Sea food	Crayfish	0.02967	0.00060	0.049733	1.274932	3.045596	1.418761	7.330361
Cereals	Sorghum	0.00317	0.00060	0.049733	2.987782	3.680654	2.156346	7.965170
	Millet	0.00195	0.00060	0.049733	4.223176	4.312190	2.737464	8.816342
	Corn	0.02017	0.00060	0.049733	1.414946	3.075503	1.472139	7.346981
	Wheat	0.00186	0.00060	0.049733	1.769614	3.175078	1.614205	7.418804
Legumes	Soybean	0.00516	0.00060	0.049733	2.311909	3.376890	1.848848	7.611337
	Beans	0.00630	0.00060	0.049733	1.311845	3.052907	1.433589	7.334147
Tubers	Cassava	0.00140	0.00060	0.049733	6.332049	5.484559	3.750929	10.560293
	Yam	0.00196	0.00060	0.049733	5.283426	4.892742	3.245050	9.664068

Table 2: Specific comminution energy E_2^*,predicted grinding time t_g, desired (300μm) and actual final product(s) size(s)

	Selected materials (samples)	Initial feed diameter (d_1) (m)	Desired (chosen) Final product diameter (d_2) (m)	E_2^* Specific Energy Requirement (kWh/kg)				t_g Predicted Crushing Time (min:sec)				Actual Final product Diameter (d_2^*) (× $10^{(-4)}$ m)			
				Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's
Sea food	Crayfish	0.02967	0.00030	0.058570	0.100493	0.073675	0.140929	02:22	04:02	02:58	05:40	5.46	3.13	3.45	3.03
Cereals	Sorghum	0.00317	0.00030	0.070443	0.111078	0.086198	0.148827	02:50	04:28	03:28	05:59	4.96	3.54	4.17	3.01
	Millet	0.00195	0.00030	0.079006	0.121603	0.096024	0.159416	03:10	04:53	03:52	06:25	4.24	3.08	3.64	3.01
	Corn	0.02017	0.00030	0.059541	0.100992	0.074627	0.141136	02:23	04:04	03:00	05:41	5.27	3.10	3.53	3.06
	Wheat	0.00186	0.00030	0.061999	0.102651	0.077030	0.142030	02:29	04:08	03:06	05:43	5.19	3.27	3.75	3.09
Legumes	Soybean	0.00516	0.00030	0.065758	0.106015	0.080998	0.144425	02:43	04:16	03:16	05:49	5.10	3.64	4.17	3.07
	Beans	0.00630	0.00030	0.058826	0.100615	0.073976	0.140977	02:22	04:03	02:58	05:40	5.27	3.34	3.66	2.97
Tubers	Cassava	0.00140	0.00030	0.093624	0.141143	0.113162	0.181113	03:46	05:41	04:33	07:17	5.68	3.23	3.58	3.14

	Yam	0.00196	0.00030	0.086355	0.131279	0.104608	0.169963	03:28	05:16	04:13	06:50	4.76	3.23	3.44	3.10

Table 3: Specific comminution energy E_2^*,predicted grinding timet_g, desired (400µm) and actual final product(s) size(s)

	Selected materials (samples)	Initial feed diameter (d_1) (m)	Desired (chosen) Final product diameter (d_2) (m)	E_2^* Specific Energy Requirement (kWh/kg)				t_g Predicted Crushing Time (min:sec)				Actual Final product Diameter (d_2^*) ($\times 10^{(-4)}$ m)			
				Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's
Sea food	Crayfish	0.02967	0.00040	0.054903	0.075113	0.062701	0.091486	02:13	03:01	02:31	03:41	4.61	4.18	4.41	4.14
Cereal	Sorghum	0.00317	0.00040	0.061848	0.080405	0.069518	0.095102	02:29	03:14	02:48	03:13	5.14	4.27	4.91	4.03
	Millet	0.00195	0.00040	0.066857	0.085668	0.074850	0.099950	02:41	03:27	03:00	04:01	4.82	4.25	4.59	4.07
	Corn	0.02017	0.00040	0.055470	0.075363	0.063240	0.091581	02:14	03:02	02:32	03:40	5.53	4.18	4.53	4.06
	Wheat	0.00186	0.00040	0.056908	0.076192	0.064544	0.091990	02:17	03:04	02:36	03:42	5.81	4.46	4.70	4.15
Legumes	Soybean	0.00516	0.00040	0.059107	0.077874	0.066697	0.093086	02:22	03:08	02:41	03:44	5.43	4.25	5.04	4.06
	Beans	0.00630	0.00040	0.055052	0.075174	0.062887	0.091508	02:13	03:01	02:32	03:41	5.53	4.26	4.44	4.16
Tubers	Cassava	0.00140	0.00040	0.075408	0.095438	0.084149	0.109883	03:02	03:50	03:23	04:25	4.92	4.39	4.41	4.02
	Yam	0.00196	0.00040	0.071156	0.090506	0.079507	0.104779	02:52	03:38	03:12	04:13	5.05	4.46	4.49	4.09

selected material to specific energy (E_2^*) obtain from each energy equation were compared. The comparative assessment revealed that the new concept energy equation termed as OruaAntia energy equation had the actual and predicted (chosen) product size(s) to be reasonably close for all the selected samples. However, this new concept energy equation capability was followed by Rittingers, bond's and then Kick's energy equations. This is evident on the average percentage deviation of 1.66, 6.88, 16.19and 38.40% respectively computed in this study and presented in Table 5.

Table 5 suggest that the new concept/approach of accomplishing any desired final product up to finer particle sizes may likely to be more reliable using OruaAntia's energy equation followed by Rittinger's equation. The percentage deviations obtain also confirm that using Kick's equation may only achieve coarse particle while coarse and intermediate particle sizes are limited to Bond's equation.Therefore, obtaining the desired coarse, intermediate and fine particles may likely be achieved

Table 4: Specific comminution energy E_2^*,predicted grinding timet_g, desired (500µm) and actual final product(s) size(s)

	Selected materials (samples)	Initial feed diameter (d_1) (m)	Desired (chosen) Final product diameter (d_2) (m)	E_2^* Specific Energy Requirement (kWh/kg)				t_g Predicted Crushing Time (min:sec)				Actual Final product Diameter (d_2^*) (× $10^{(-4)}$ m)			
				Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's	Kick's	Rittinger's	Bond's	OruaAntia's
Sea food	Crayfish	0.02967	0.00050	0.052058	0.059885	0.055212	0.065421	02:05	02:25	02:13	02:38	5.88	5.11	5.48	5.08
Cereals	Sorghum	0.00317	0.00050	0.055181	0.062002	0.058136	0.066780	02:13	02:29	02:20	02:41	5.70	5.13	5.32	5.01
	Millet	0.00195	0.00050	0.057433	0.064107	0.060400	0.068601	02:19	02:35	02:26	02:46	5.59	5.25	5.37	5.06
	Corn	0.02017	0.00050	0.052313	0.059985	0.055470	0.065457	02:06	02:25	02:14	02:38	5.89	5.11	5.60	5.08
	Wheat	0.00186	0.00050	0.052960	0.060317	0.056023	0.065611	02:08	02:26	02:15	02:38	5.90	5.30	5.64	5.01
Legumes	Soybean	0.00516	0.00050	0.053948	0.060990	0.056937	0.066023	02:10	02:27	02:17	02:40	5.58	5.21	5.32	5.10
	Beans	0.00630	0.00050	0.052125	0.059910	0.055319	0.065429	02:06	02:25	02:13	02:38	5.89	5.23	5.56	5.03
Tubers	Cassava	0.00140	0.00050	0.061278	0.068015	0.064349	0.072334	02:28	02:44	02:35	02:55	5.62	5.18	5.51	5.04
	Yam	0.00196	0.00050	0.059366	0.066042	0.062378	0.070416	02:23	02:40	02:31	02:50	5.69	5.12	5.51	5.06

Table 5: % Deviation of Actual Final average diameter d_2^* with predicted (chosen) final average diameter d_2 of selected materials subjected to comminution

		Diameter of selected materials (m)										
		Crayfish ($10^{(-4)}$)	Sorghum ($10^{(-4)}$)	Millet ($10^{(-4)}$)	Corn ($10^{(-4)}$)	Wheat ($10^{(-4)}$)	Soybean ($10^{(-4)}$)	Beans ($10^{(-4)}$)	Cassava ($10^{(-4)}$)	Yam ($10^{(-4)}$)	$\sum d_2^*-d_2$ ($10^{(-4)}$)	E_p
Kick's equation	d_2^*	5.46	4.96	4.24	5.27	5.19	5.10	5.27	5.68	4.76		
	d_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00		
	$d_2^*-d_2$	2.46	1.96	1.24	2.27	2.19	2.10	2.27	2.68	1.76	18.93	70.11
	d_2^*	4.61	5.14	4.82	5.53	5.81	5.43	5.53	4.92	5.05		
	d_2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00		

	$d_2^*-d_2$	0.61	1.14	0.82	1.53	1.81	1.43	1.53	0.92	1.05	10.84	30.11
	d_2^*	5.88	5.70	5.59	5.89	5.90	5.58	5.89	5.62	5.69		
	d_2	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00		
	$d_2^*-d_2$	0.88	0.70	0.59	0.89	0.90	0.58	0.89	0.62	0.69	6.74	14.99
	E_{pa}											38.40
Rittinger's equation	d_2^*	3.13	3.54	3.08	3.10	3.27	3.64	3.34	3.23	3.23		
	d_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00		
	$d_2^*-d_2$	0.13	0.54	0.08	0.10	0.27	0.64	0.34	0.23	0.23	2.56	9.49
	d_2^*	4.18	4.27	4.25	4.18	4.46	4.25	4.26	4.39	4.46		
	d_2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00		
	$d_2^*-d_2$	0.18	0.27	0.25	0.18	0.46	0.25	0.26	0.39	0.46	2.70	7.50
	d_2^*	5.11	5.13	5.25	5.11	5.30	5.21	5.23	5.18	5.12		
	d_2	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00		
	$d_2^*-d_2$	0.11	0.13	0.25	0.11	0.30	0.21	0.23	0.18	0.12	1.64	3.64
	E_{pa}											6.88
Bond's equation	d_2^*	3.45	4.17	3.64	3.53	3.75	4.17	3.66	3.58	3.44		
	d_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00		
	$d_2^*-d_2$	0.45	1.17	0.64	0.53	0.75	1.17	0.66	0.58	0.44	6.39	23.67
	d_2^*	4.41	4.91	4.59	4.53	4.70	5.04	4.44	4.41	4.49		
	d_2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00		
	$d_2^*-d_2$	0.41	0.91	0.59	0.53	0.70	1.04	0.44	0.41	0.49	5.52	15.33
	d_2^*	5.48	5.32	5.37	5.60	5.64	5.32	5.56	5.51	5.51		
	d_2	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00		
	$d_2^*-d_2$	0.48	0.32	0.37	0.60	0.64	0.32	0.56	0.51	0.51	4.31	9.58
	E_{pa}											16.19
OruaAntia's equation	d_2^*	3.03	3.01	3.01	3.06	3.09	3.07	2.97	3.14	3.10		
	d_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00		
	$d_2^*-d_2$	0.03	0.01	0.01	0.06	0.09	0.07	-0.03	0.14	0.10	0.48	1.78
	d_2^*	4.14	4.03	4.07	4.06	4.15	4.06	4.16	4.02	4.09		
	d_2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00		
	$d_2^*-d_2$	0.14	0.03	0.07	0.06	0.15	0.06	0.16	0.02	0.09	0.78	2.17
	d_2^*	5.08	5.01	5.06	5.08	5.01	5.10	5.03	5.04	5.06		
	d_2	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00		
	$d_2^*-d_2$	0.08	0.01	0.06	0.08	0.01	0.10	0.03	0.04	0.06	0.47	1.04
	E_{pa}											1.66

using Orua Antia's and Rittinger's equations but much finer particles may possibly be further achieved using the Orua Antia's energy equation.

CONCLUSION

Predicting and achieving closely the desired final particle size following comminution of an initial particle size could be achieved more closely using the OruaAntia's energy equation for mass-size reduction operation followed by Rittinger's energy equation. The computed average percentage deviation 38.40%, 16.19%, 6.88% and 1.66% of the desired particle size(s) with the actual particle size(s) of material subjected to comminution re-affirm the fact that kick's equation may be limited only to coarse particle size reduction; while Bond's energy equation may be for both coarse and intermediate particle size prediction. Moreso, the likely prediction of coarse, intermediate and fine particles could be achieved using Rittinger's and OruaAntia's energy equations. However, further size reduction to finer particle sizes may be better achieved using the OruaAntia's minimum energy equation which is a new concept/approach for mass-size reduction operations of a given material.

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