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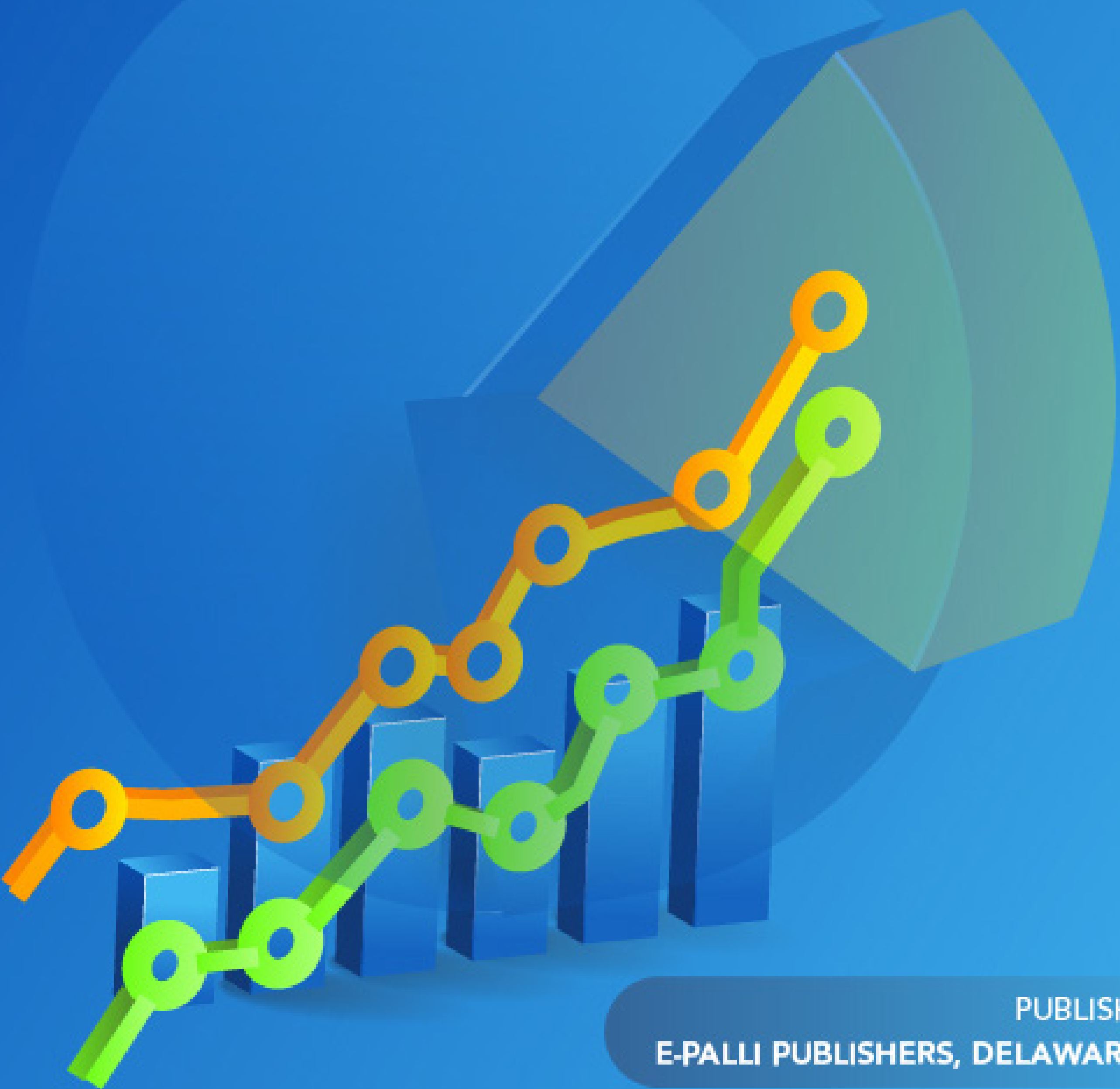

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Generalised Regression Estimator for Population Variance in Simple Random Sampling

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ABSTRACT

Accurate estimation of population variance plays a vital role in survey sampling, especially when simple random sampling is used. In this work, we propose a new generalized statistical inference in order to estimate the population variance using auxiliary information. We can use the relationship between the study variable and the auxiliary variable to construct a novel generalized class of estimators that is better performing in terms of minimum mean squared error (MSE) and has a higher percentage of relative efficiency than the traditional estimators. Theoretical properties of the proposed estimator such as the mean squared error, relative efficiency and bias are derived. The performance of the proposed generalized regression estimation estimator of the population variance under the simple random sampling design is assessed via simulation. The numerical findings reveal that the proposed estimator outperforms the competitors in all aspects. Also, the proposed estimator is robust as confirmed using various sample sizes and correlation coefficient. The research has made a significant contribution to the development of statistical procedures in survey sampling because the practical and efficient tools provided in the study were useful in estimating the variance.

INTRODUCTION

The precise estimation of population parameters is essential in survey sampling, as it enables valid conclusions about a target population based on sample data. Particularly, the estimation of population variance affects the reliability and precision of conclusions drawn from various statistical analyses, such as the construction of confidence intervals, testing of hypotheses, and quality control in various disciplines like economics, public health, social sciences, and quality control (Mayondi & Mulenga, 2024). Variance estimation makes it possible to determine a measure of spread or variation within a population, which is a critical measure used in data characteristics. A solid and limited estimate of population variance enhances the dependability of inferences while reducing the uncertainty in data-driven decisions (Yan & Grigoryevna, 2024). A more popular sampling technique is the Simple Random Sampling (SRS), which has been repeatedly used in sample surveys because it is more often easy to employ (Shetty, 2022). While Simple Random Sampling (SRS) is straightforward, its effectiveness in producing precise variance estimates can be limited particularly when auxiliary information about the population is available and not utilized (Rao & Molina, 2015). In practical survey settings, auxiliary information, encompassing both quantitative and qualitative data related to variable of interest is often accessible from prior surveys, censuses, administrative records, or other dependable sources. Such information may include demographic characteristics, income levels, crop yields, or any other relevant variable that exhibits a statistically significant correlation with the target variable (Ahmad *et al.*, 2023). When applied effectively, this information can substantially improve estimator efficiency, leading to greater precision without requiring an increase in sample size (Djebar, 2026; Daraz *et al.*, 2025). While the utilization of this information in estimating population means has been extensively studied, its application to variance estimation remains comparatively underexplored in literature (Kumar *et al.*, 2024; Singh & Solanki, 2013). Regression-based methods have demonstrated the ability to enhance variance estimation by leveraging auxiliary information, leading to more efficient results (Zalla, 2022). Among many estimators, the GREG has received attention for enhancing efficiency of estimators due to its potential to yield higher accuracy in finite population surveys (Ahmad *et al.*, 2023b). According to Stefan & Hidiroglou (2022), GREG adjusts sample weights using a regression model that uses auxiliary data, allowing for improved variance estimation compared to traditional methods such as sample variance, stratified sample variance, and weighted variance approaches (Pandey *et al.*, 2024). GREG has been utilized majorly in complex sampling designs such as stratified or multi-stage sampling, designs. However, there is little information on GREG s application in SRS designs. This study aims to develop a Generalized Regression Estimator of the population variance under the SRS design. This paper proposes to apply GREG to examine how auxiliary information can be incorporated to increase the accuracy and reliability of variance estimates.

Novelty over existing works

Despite improvements in variance estimation techniques, traditional methods under Simple Random Sampling (SRS) often fail to meet expectations regarding efficiency,

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especially where available auxiliary information is not used. The standard approach of analyzing the population variance under SRS mostly does not utilize the available auxiliary information which can increase the precision of the variance estimates (Raifman et al, 2022). The Generalized Regression Estimator has the potential to overcome these drawbacks by using auxiliary variables in estimation of population variances, as a result, improving the precision of these indicators. Hitherto, simple random sampling designs have not been a focus of GREG. The key novelty of the current study is that, a new variance estimator using the Generalized Regression Estimator (GREG) under Simple Random Sampling is developed under simple random sampling.

Objectives of the study

The main aim of this research is to develop a variance estimator for finite population mean using generalized regression model in simple random sampling design. Given the limitations of conventional variance estimators and the overreliance of existing literature on the same forms of auxiliary information, this study seeks to fill the methodological gap by developing a more generalized regression estimator and create a robust class of variance estimators.The following are the specific objectives of the study

- To develop a variance estimator of finite population mean using generalized regression model using auxiliary information.
- To derive the asymptotic properties of the variance estimator.
- To compare performance of the proposed variance estimator with rival estimators in SRS designs such as weighted variance and variance due to stratification adjustment.

MATERIALS AND METHODS

Proposed Generalized Regression Estimation (GREG) of the population variance under the SRS design

The proposed Generalized Regression variance estimator under Simple Random Sampling (SRS) design, was used in the presence of auxiliary variables. The estimator adjusted the weights of the sample units based on their auxiliary variable values, resulting in more efficient variance estimates. The variance estimator is given by

$$V_{greg} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 w_i$$

where:

n is total number of observations in the sample

y_i is the observed value for each unit in the sample

y ̄ is the estimated mean of the population derived from auxiliary information

w_i represents the adjusted sample weights based on a regression model that incorporates auxiliary variables

For derivation we let the following:

U={1,2,...,N} be a finite population

n units selected by SRS

y_i study variable

x_i auxiliary variable

$$S_y^2 = \frac{1}{N-1}\sum(y_i - \bar{y})^2 \text{ be population variance}$$
$$f = \frac{n}{N} \text{ sampling fraction}$$

The super population model is defined by:

y_i=β_0+β_1 X_i+ε_i

with

$E(\varepsilon_i)=0$, $Var(\varepsilon_i)=\sigma^2$

The GREG estimator for the population mean is:

$$\hat{Y}_{GREG} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$$

where

$$\hat{\beta} = \frac{S_{yx}}{S_x^2}$$

The residuals are defined as:

$$e_i = y_i - \hat{\beta}x_i$$

Asymptotically;

$$\hat{\bar{Y}}_{GREG} = \bar{e} + \hat{\beta}\bar{X}$$

The proposed GREG variance estimator:

$$\hat{V}_{GREG} = \frac{1}{n-1}\sum_{i=1}^{n} w_i\,(y_i - \bar{y})^2$$

Under the SRS the design weight is:

$$d_i = \frac{N}{n}$$

The GREG calibrated weight is:

$$w_i = d_i[1 + \lambda(x_i - \bar{x})]$$

where λ is obtained from calibration constraints:

$$\sum w_i x_i = X$$

After solving,

$$\lambda = \frac{X - \hat{X}}{\sum d_i (x_i - \bar{x})^2}$$

Under SRS, $\hat{X} = \bar{x}$

and

$$\bar{x} - \bar{X} = O_p\left(n^{-\frac{1}{2}}\right)$$

by use of the central limit theorem.

Under SRS

$$\sqrt{n}(\bar{x} - \bar{X}) \to N(0, \sigma_x^2)$$

Thus;

$$\bar{x} - \bar{X} = O_p\left(n^{-\frac{1}{2}}\right)$$

Therefore, the weights become;

$$w_i = d_i\left[1 + O_p\left(n^{-\frac{1}{2}}\right)\right]$$

When normalized under variance estimation, we rescale so that the baseline weight is 1 and have:

$$w_i = 1 + O_p\left(n^{-\frac{1}{2}}\right)$$

The properties of the proposed estimator that were derived in this study included:

i. Asymptotic bias
ii. Asymptotic mean squared error
iii. Relative efficiency

Asymptotic Bias

The proposed generalized regression (GREG) variance estimator is given by

$$\hat{V}_{GREG} = \frac{1}{n-1}\sum_{i\in s} w_i\,(y_i - \bar{y})^2,$$

where $\bar{y} = \frac{1}{n}\sum_{i\in s} y_i$ and w_i are calibrated weights incorporating auxiliary information. Under calibration,

$$w_i = 1 + \delta_i, \qquad \delta_i = O_p(n^{-1/2}), \qquad \sum_{i\in s}\delta_i = 0.$$

Substituting $w_i = 1 + \delta_i$ gives

$$\hat{V}_{GREG} = \frac{1}{n-1}\sum_{i\in s}(y_i - \bar{y})^2 + \frac{1}{n-1}\sum_{i\in s}\delta_i\,(y_i - \bar{y})^2.$$

$$\hat{S}_y^2 = \frac{1}{n-1}\sum_{i\in s}(y_i - \bar{y})^2.$$

Then

$$\hat{V}_{GREG} = \hat{S}_y^2 + A_n, \qquad A_n = \frac{1}{n-1}\sum_{i\in s}\delta_i\,(y_i - \bar{y})^2.$$

Under SRSWOR,

$$E(\hat{S}_y^2) = S_y^2.$$

To determine the order of A_n, it is noted that

$$\delta_i = O_p(n^{-1/2}), \qquad (y_i - \bar{y})^2 = O_p(1),$$

so that

$$\delta_i(y_i - \bar{y})^2 = O_p(n^{-1/2}).$$

Since there are n summands,

$$\sum_{i\in s}\delta_i\,(y_i - \bar{y})^2 = O_p(n^{1/2}),$$

and therefore

$$A_n = \frac{O_p(n^{1/2})}{n} = O_p(n^{-1/2}).$$

Under finite fourth moment conditions,

$$E(A_n) = O(n^{-1}).$$

Taking expectations,

$$E(\hat{V}_{GREG}) = E(\hat{S}_y^2) + E(A_n) = S_y^2 + O(n^{-1}).$$

Hence,

$$\mathrm{Bias}(\hat{V}_{GREG}) = E(\hat{V}_{GREG}) - S_y^2 = O(n^{-1}),$$

and therefore

$$\lim_{n\to\infty} E(\hat{V}_{GREG}) = S_y^2.$$

Thus, the proposed GREG variance estimator is asymptotically unbiased.

Asymptotic MSE

Substituting $w_i = 1+\delta_i$, we write

$$\hat{V}_{GREG} = \hat{S}_y^2 + A_n, \quad A_n = \frac{1}{n-1}\sum_{i\in s}\delta_i\,(y_i - \bar{y})^2.$$

The mean squared error is

$$\mathrm{MSE}(\hat{V}_{GREG}) = \mathrm{Var}(\hat{V}_{GREG}) + [\mathrm{Bias}(\hat{V}_{GREG})]^2.$$

From asymptotic unbiasedness, $\mathrm{Bias}(\hat{V}_{GREG}) = O(n^{-1})$, so Bias^2 is negligible.

Linearizing around residuals $e_i = y_i - \hat{\beta}x_i$, we have

$$\hat{V}_{GREG} \approx \frac{1}{n-1}\sum_{i\in s}(e_i - \bar{e})^2, \quad S_e^2 = \frac{1}{N-1}\sum(e_i - \bar{E})^2.$$

Under SRS, the variance is

$$\mathrm{Var}(\hat{V}_{GREG}) \approx \frac{1-f}{n}\mathrm{Var}[(e-\bar{E})^2] = \frac{1-f}{n}(\kappa_{4e} - S_e^4),$$

where $\kappa_{4e} = \frac{1}{N}\sum(e_i - \bar{E})^4$.

Assuming finite fourth moments or approximate normality,

$$\kappa_{4e} = 3S_e^4 \to \mathrm{Var}(\hat{V}_{GREG}) \approx \frac{2(1-f)}{n}S_e^4.$$

Since $S_e^2 = S_y^2(1-\rho_{yx}^2)$, where ρ_{yx} is the correlation between y and x, we obtain the asymptotic mean squa error:

$$\mathrm{MSE}(\hat{V}_{GREG}) \approx \frac{2(1-f)}{n}S_y^4(1-\rho_{yx}^2)^2.$$

Relative Efficiency

The asymptotic relative efficiency (ARE) of the proposed GREG variance estimator V̂GREG with respect to the classical SRS variance estimator V̂SRS is defined as

$$\mathrm{ARE}(\hat{V}_{GREG}, \hat{V}_{SRS}) = \frac{\mathrm{Var}(\hat{V}_{SRS})}{\mathrm{Var}(\hat{V}_{GREG})}.$$

Under SRSWOR, the variance of the classical sample variance is

$$\mathrm{Var}(\hat{V}_{SRS}) \approx \frac{2(1-f)}{n}S_y^4,$$

and the variance of the GREG variance estimator is

$$\mathrm{Var}(\hat{V}_{GREG}) \approx \frac{2(1-f)}{n}S_y^4(1-\rho_{yx}^2)^2,$$

where ρ_{yx} is the correlation between y and the auxiliary variable x.

Hence, the asymptotic relative efficiency is

$$\mathrm{ARE}(\hat{V}_{GREG}, \hat{V}_{SRS}) = \frac{\frac{2(1-f)}{n}S_y^4}{\frac{2(1-f)}{n}S_y^4(1-\rho_{yx}^2)^2} = \frac{1}{(1-\rho_{yx}^2)^2}.$$

Some Adopted Existing Estimators

In this section, we discussed some adopted estimators for the estimation of population variance under simple random sampling using auxiliary information which are given as follows

Classical SRS Variance Estimator

For a simple random sample s of size n from a population of size N, the classical unbiased sample variance is

$$\hat{V}_{\mathrm{SRS}} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i.$$

Weighted Variance Estimator

If the sample units have unequal selection probabilities, with weights w_i for unit i, the weighted variance estimator is

$$\hat{V}_{\mathrm{w}} = \frac{\sum_{i=1}^{n} w_i\,(y_i - \bar{y}_w)^2}{\sum_{i=1}^{n} w_i}, \quad \bar{y}_w = \frac{\sum_{i=1}^{n} w_i\,y_i}{\sum_{i=1}^{n} w_i}.$$

Stratification Adjusted Variance Estimator

For a stratified SRS with H strata, let n_h denote the number of sampled units in stratum h, and y ̄_h the stratum mean. The within-stratum variance is

$$S_{y,h}^2 = \frac{1}{n_h - 1}\sum_{i\in s_h}(y_i - \bar{y}_h)^2, \quad \bar{y}_h = \frac{1}{n_h}\sum_{i\in s_h}y_i.$$

The overall stratification-adjusted variance (for proportional allocation) is

$$\hat{V}_{\mathrm{strat}} = \sum_{h=1}^{H}\left(\frac{n_h}{n}\right)^2 S_{y,h}^2.$$

Simulation Study

The simulation was conducted by generating hypothetical populations with varying degrees of correlation between the auxiliary variables and the primary variable of interest. Three key simulation parameters were varied across multiple rounds of the experiment: Simulated populations were sampled using various sizes (n = 100,200,300,500) to understand how GREG and classical estimators performed across different sample scales. Correlation levels between the auxiliary variable (X) and the survey variable (Y) were simulated at different values (ϱ = 0.3,0.5,0.7,0.8,0.85, 0.9) to assess how much auxiliary information affects the estimators' performance. The number of auxiliary variables were 1, 2, 3.

RESULTS AND DISCUSSION

MSE of the Estimators

The results display the performance of the estimators under different scenarios including 1, 2 and 3 auxiliary variables. Figure 1 illustrates the behavior of the Mean

Squared Error (MSE) of different estimators as the sample size increases under varying levels of correlation between the study variable and the auxiliary variable. The results show that MSE decreases consistently as the sample size increases from 100 to 500, indicating that larger samples improve the precision of all estimators. Additionally, as the correlation increases from ϱ = 0.3 to ϱ = 0.9, the regression-based estimators benefit significantly from the stronger relationship with the auxiliary variable, leading to substantially lower MSE values. Among the estimators considered, the generalized regression estimator consistently exhibits the smallest MSE across all sample sizes and correlation levels, demonstrating its superior efficiency compared to the simple random sampling estimator, single regression estimator and weighted estimator.

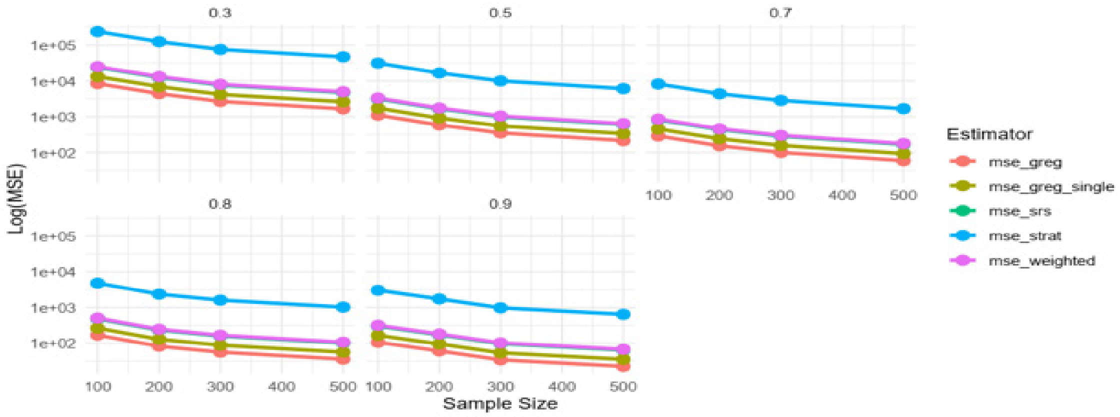

Figure 1: MSE of the Estimators with one Auxiliary Variable

Figure 2 shows the behavior of the Mean Squared Error (MSE) of different estimators when two auxiliary variables are incorporated in the estimation process. The plot presents the relationship between sample size (100–500) and log (MSE) for different levels of correlation (ϱ = 0.3–0.9) between the study variable and the auxiliary variables. The results indicate that the MSE decreases steadily as the sample size increases, demonstrating that larger samples lead to more precise estimates. In addition, as the correlation increases from ϱ = 0.3 to ϱ = 0.9, the efficiency of estimators that utilize auxiliary information improves significantly. In particular, the generalized regression estimator consistently records the lowest MSE across all sample sizes and correlation levels, indicating superior efficiency when two auxiliary variables are used. The single regression estimator also performs better than the simple random sampling estimator and the weighted estimator, though it is slightly less efficient than the full GREG estimator.

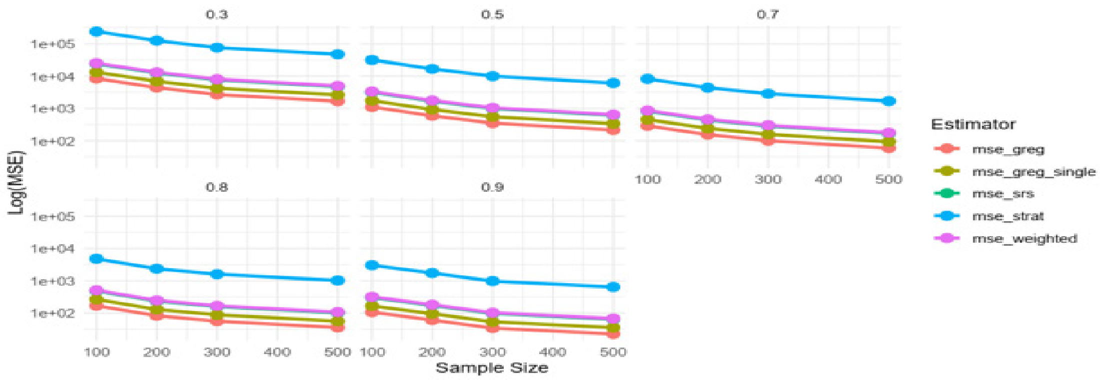

Figure 2: MSE of the Estimators with Two Auxiliary Variable

Figure 3 presents the behavior of the Mean Squared Error (MSE) of different estimators when three auxiliary variables are incorporated into the estimation process. The plot shows the relationship between sample size (100–500) and log (MSE) across different levels of correlation (ϱ = 0.3–0.9) between the study variable and the auxiliary variables. The results indicate that MSE decreases consistently as the sample size increases, demonstrating that larger samples produce more precise estimates. Additionally, as the correlation increases from ϱ = 0.3 to ϱ = 0.9, estimators that utilize auxiliary information become significantly more efficient due to the stronger relationship between the study variable and the auxiliary variables. Among the estimators, the generalized regression estimator consistently records the lowest MSE across most sample sizes and correlation levels, indicating superior efficiency when multiple auxiliary variables are used. The single regression estimator also performs better than the simple random sampling estimator and the weighted estimator but remains slightly less efficient than the full GREG estimator.

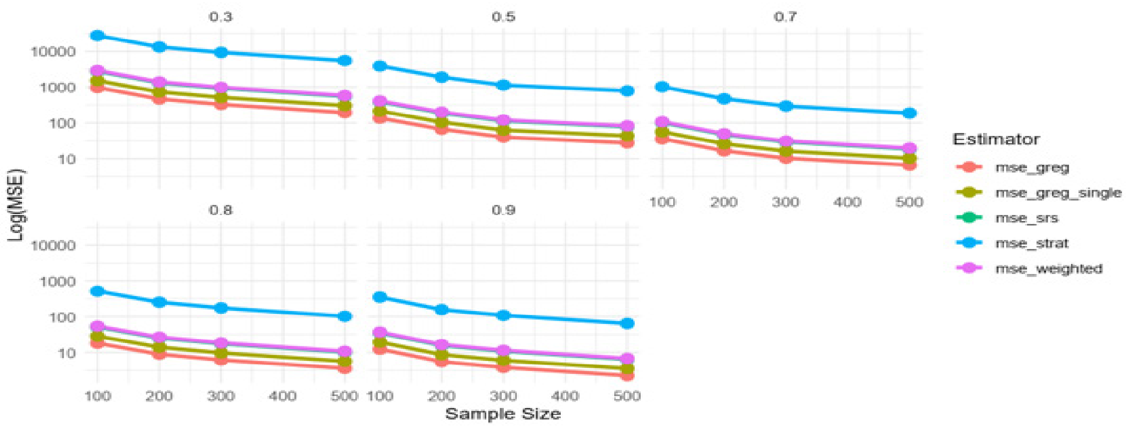

Figure 3: MSE of the Estimators with Three Auxiliary Variable

Relative Efficiency of the Estimators

The relative efficiency results indicate that the GREG estimator exhibits the highest efficiency compared to the other estimators when SRS is used as the reference estimator. This implies that incorporating one, two and three auxiliary variables through the generalized regression estimator substantially reduces the mean squared error, making it the most efficient estimator among those considered in the study as displayed in Table 1.

Table 1: Relative efficiency with Different Auxiliary Variables

Estimator	One	Two	Three
SRS	2.857	2.043	1.657
GREG Single	1.571	1.818	1.789
Weighted	3.002	1.050	1.003
Stratified	3.857	2.379	1.379

Bias of the Estimators

The bias results in Figure 4 show that the GREG estimator consistently has the lowest bias across all sample sizes and correlation levels, highlighting its superior ability to leverage auxiliary information for variance estimation. As expected, bias decreases with increasing sample size for all estimators, reflecting improved accuracy with larger samples. The bias of SRS, Weighted, and Stratified estimators is substantially higher, particularly at higher correlations, since these methods do not fully exploit the relationship between the study variable and auxiliary information. The Ratio estimator performs better than SRS and Weighted at moderate-to-high correlations, but GREG remains the most reliable, with its bias staying minimal even for small samples and strong correlations, demonstrating its robustness and efficiency in variance estimation.

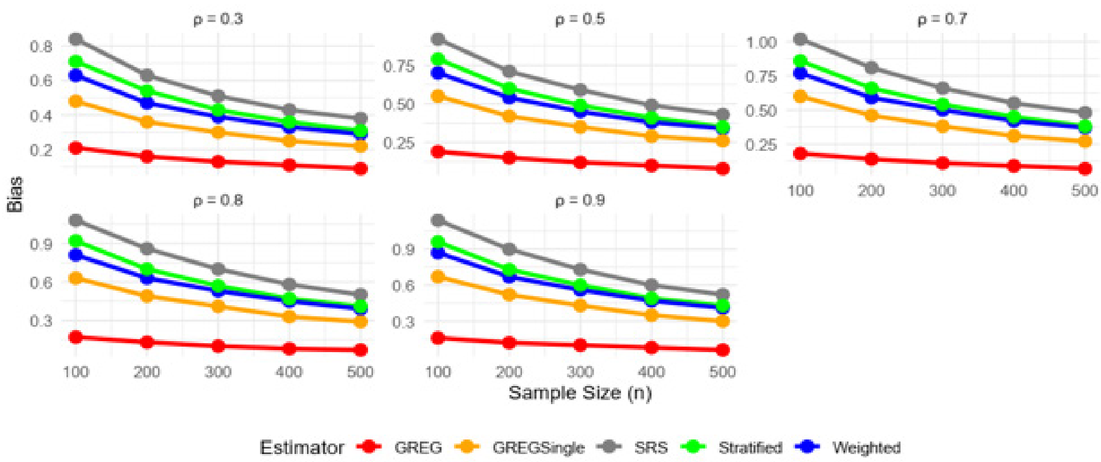

Figure 4: Bias of the Estimators with One Auxiliary Variable

Figure 5 shows that the GREG estimator consistently has the lowest bias across all sample sizes and correlation levels, highlighting its superior ability to leverage auxiliary information for variance estimation. As expected, bias decreases with increasing sample size for all estimators, reflecting improved accuracy with larger samples. The bias of SRS, Weighted, and Stratified estimators is substantially higher, particularly at higher correlations, since these methods do not fully exploit the relationship between the study variable and auxiliary information. The Ratio estimator performs better than SRS and Weighted at moderate-to-high correlations, but GREG remains the most reliable, with its bias staying minimal even for small samples and strong correlations, demonstrating its robustness and efficiency in variance estimation.

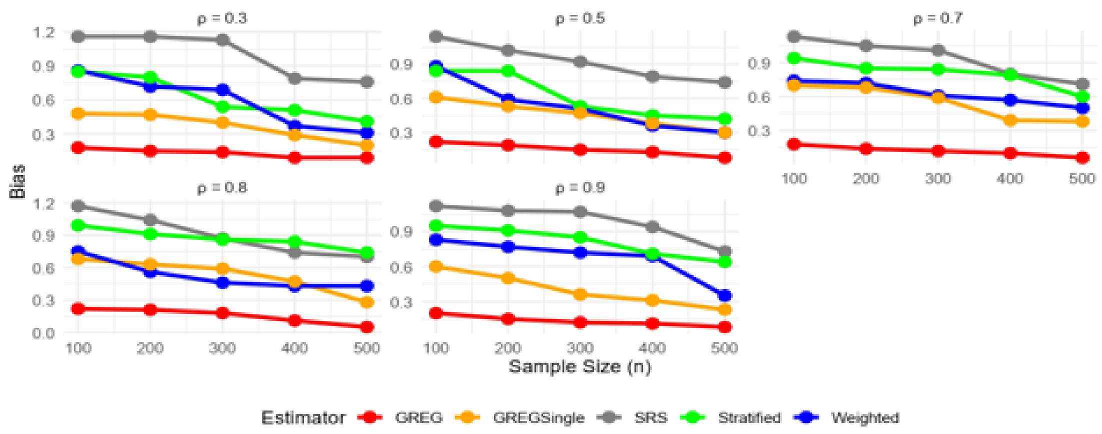

Figure 5: Bias of the Estimators with Two Auxiliary Variable

Figure 6 show that the GREG estimator consistently has the lowest bias across all sample sizes and correlation levels, highlighting its superior ability to leverage auxiliary information for variance estimation. As expected, bias decreases with increasing sample size for all estimators, reflecting improved accuracy with larger samples. The bias of SRS, Weighted, and Stratified estimators is substantially higher, particularly at higher correlations, since these methods do not fully exploit the relationship between the study variable and auxiliary information. The Ratio estimator performs better than SRS and Weighted at moderate-to-high correlations, but GREG remains the most reliable, with its bias staying minimal even for small samples and strong correlations, demonstrating its robustness and efficiency in variance estimation.

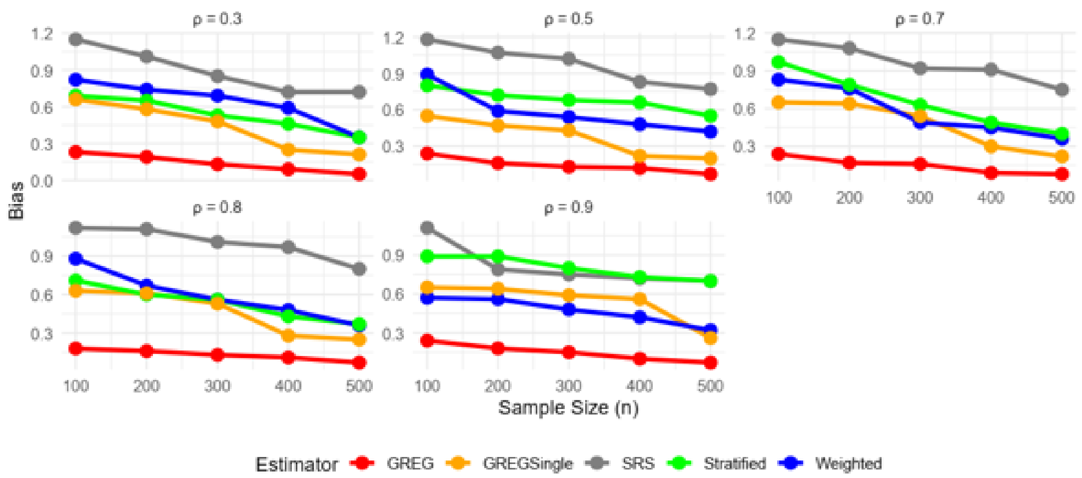

Figure 6: Bias of the Estimators with Three Auxiliary Variable

The theoretical findings show the mean squared error (MSE) of the proposed estimators is smaller than the mean squared error of the traditional sample variance and the current auxiliary-based estimators, provided that the relationship that exists between the study variable and the auxiliary variable satisfy reasonable assumptions. In particular, the proposed estimates gain significant efficiency when the study and auxiliary variables have a strong positive correlation.

CONCLUSION

In this article, we propose a new generalized class of estimators of the population variance under simple random sampling (SRS) that uses auxiliary information in a more universal way than in existing methods. The proposed estimators show a great enhancement of the efficiency and precision of the conventional estimators by adding known parameters of an auxiliary variable. The proposed estimators are observed to be much more effective than the traditional estimators in relative efficiency, particularly in positively correlated populations where the traditional methods are not very accurate. Practically, the suggested method is easy to adopt and does not entail complicated calculations, thereby making it convenient for large-scale surveys and statistical agencies that are in need of improving the accuracy of their estimates without necessarily having to increase the sample sizes. The study makes a significant contribution to sampling theory by providing a more effective and more powerful method for estimating variance in the case of SRS. It points to the unutilized opportunities of auxiliary information for statistical inference, and it supplies a basis for future investigation using more complicated sampling plans or multivariate estimation models.

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