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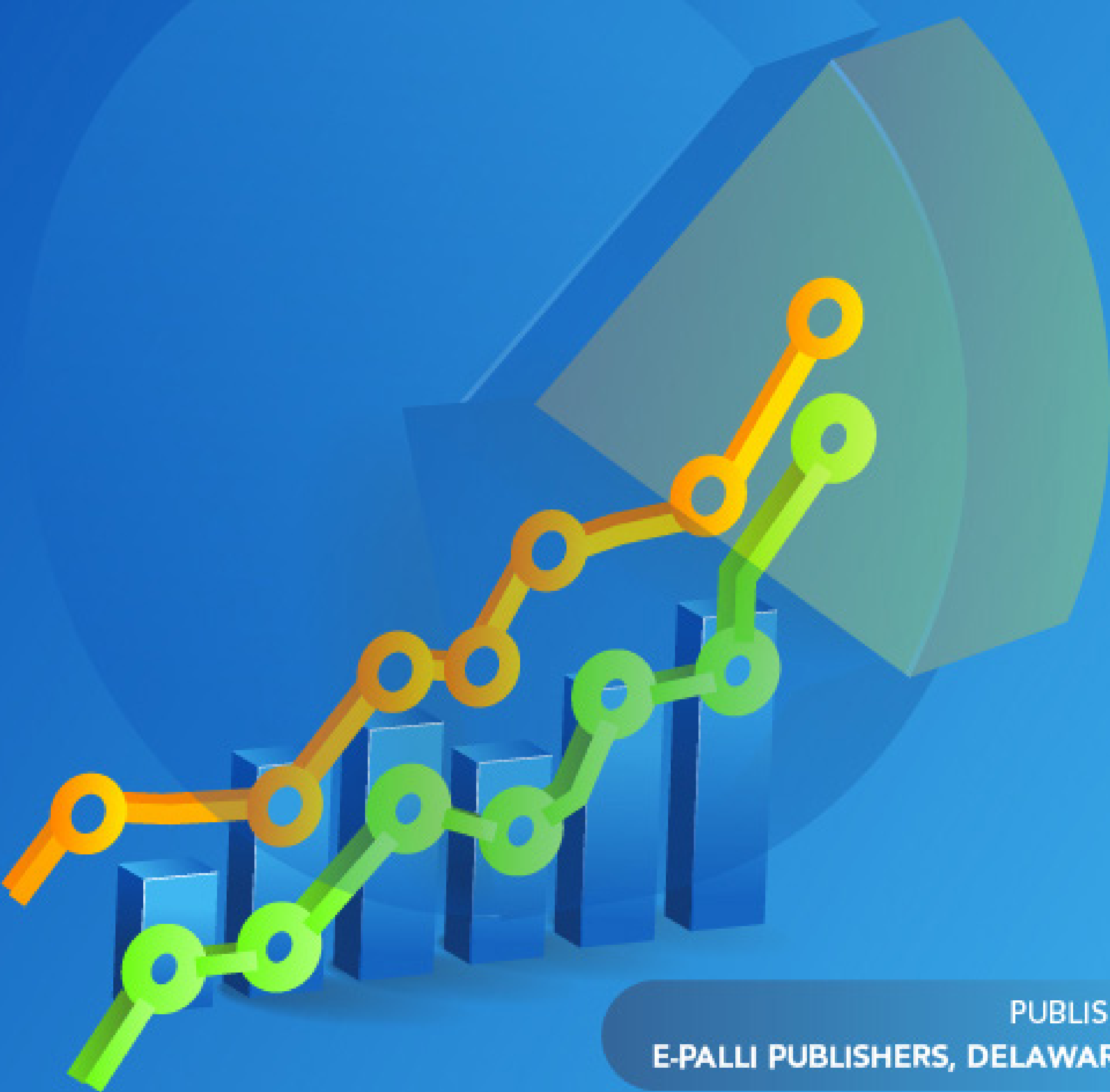

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# Simultaneously Addressing Collinearity and Outliers: A Robust Two-Parameter Estimation Approach

Seno Ofonmbuk Albert[1*], Samuel Olayemi Olanrewaju[1], Emmanuel Segun Oguntade[1]

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## ABSTRACT

Multicollinearity and outliers remain two major challenges in linear regression modeling, often occurring simultaneously in practical applications and leading to instability, inflated variance, and unreliable inference. Although shrinkage estimators such as ridge and Liu estimators effectively address multicollinearity, but are still sensitive to outliers. Conversely, robust estimators mitigate the influence of outliers but do not adequately resolve collinearity. This study proposes a new robust two-parameter shrinkage estimator (Rprop1) designed to simultaneously handle multicollinearity and outliers within the classical linear regression framework. The estimator is derived in canonical form, and its statistical properties are established through mean squared error (MSE) analysis. A comprehensive Monte Carlo simulation study is conducted across varying sample sizes, error variances, correlation levels, numbers of explanatory variables, and outlier magnitudes. The performance of the proposed estimator is compared with existing robust and shrinkage-based estimators, including robust ridge, robust Liu, and robust Kibra Lukman estimators. Simulation results consistently demonstrate that the proposed estimator achieves superior MSE performance across moderate to severe multicollinearity and outliers' magnitudes and it's also supported by the real life dataset. The findings suggest that the proposed method provides a more stable and efficient alternative for regression modeling in the presence of simultaneous collinearity and outliers.

## INTRODUCTION

Regression analysis constitutes a principal framework for examining the relationship between a response variable Y and a set of explanatory variables within a linear modeling structure. Under the classical linear regression assumptions, the Ordinary Least Squares (OLS) estimator is the Best Linear Unbiased Estimator (BLUE) and achieves minimum variance among linear unbiased estimators (Gujarati, 2003; Oladapo et al., 2023). A fundamental requirement for these properties is the absence of exact linear dependence among the explanatory variables, that is, the non-existence of multicollinearity. In practical data environments, particularly in observational and high-dimensional settings, multicollinearity frequently arises. Its presence is commonly assessed using diagnostic measures such as the Variance Inflation Factor (VIF). Severe multicollinearity inflates the variances of parameter estimates, reduces statistical precision, and may produce regression coefficients with implausible magnitudes or signs, thereby undermining inferential reliability (Gujarati, 2003; Idowu et al., 2023). To mitigate this problem, several biased estimators have been introduced. Notable among these are the Stein estimator (Stein, 1956), the ridge estimator proposed by Hoerl and Kennard (1970), and the Liu estimator (Liu, 1993). These approaches deliberately introduce bias to achieve variance reduction and improved mean squared error performance. Contemporary refinements and empirical applications have further demonstrated the practical value of such biased estimation strategies (Akdeniz & Kaciranlar, 1995; Oladapo et al., 2023; Idowu et al., 2022; Owolabi et al., 2022 Baldawire et al., 2023).

Beyond multicollinearity, regression modeling is often compromised by outliers and leverage points. Outliers correspond to atypical response values, whereas leverage points arise from extreme configurations of explanatory variables. Diagnostic measures such as studentised deleted residuals and Mahalanobis distance are widely applied to identify such observations, while influence statistics including Cook's distance and DFFITS quantify their impact on parameter estimates. Robust regression methodologies have therefore been developed to limit the influence of anomalous observations. Prominent examples include the M-estimator (Huber, 1964), Least Trimmed Squares (Rousseeuw & Van Driessen, 1998), the S-estimator (Rousseeuw & Yohai, 1984), and the MM-estimator (Yohai, 1987).In many applied contexts, multicollinearity and outliers coexist, producing a compounded estimation challenge that cannot be effectively resolved by methods designed to address either problem in isolation. This dual occurrence has motivated the development of hybrid estimators that integrate biased and robust estimation principles. Holland (1973) introduced a robust M-estimator within a ridge framework. Askin and Montgomery (1980) extended this idea, while Walker (1984) incorporated Generalized M-estimators. Simpson and Montgomery (1996) further proposed a multistage Generalized M-estimation combined with iterated ridge regression. Similarly, Arslan and Billor (2000) developed the Liu M-estimator to enhance robustness against outliers. Also the Robust Kibra–Lukman (RKL) estimator and modification (Majid et al., 2022) have contributed to this evolving class of hybrid estimators. Lukman et al. (2014)



[1] Department of Statistics, Faculty of Science, University of Abuja, Abuja, Nigeria
[*] Corresponding author's e-mail: senosuzie@gmail.com

proposed and applied some Robust Ridge Regression Estimators to the Hussein and Abdalla (2012) data. Alpu and Samkar (2010) applied the Liu estimator based on the M estimator to VO2 data, Adejumo et al. (2024), Fayose et al. (2023), Awad et al. (2021), Dawoud and Abonazel (2021), and many others also worked extensively on Robust estimators.

Motivated by these developments, this study advances the theoretical and empirical investigation of robust biased estimation within the linear regression framework. The objective is to formulate and evaluate an estimator that simultaneously accommodates multicollinearity and outliers, and to compare its performance with established hybrid estimators under controlled statistical criteria.

## MATERIALS AND METHODS

### Linear Regression Model and OLS Estimator

Consider the standard regression model:

Consider the standard regression model:

$$y = X\beta + \varepsilon \qquad (1)$$

where y is an n×1 vector of the dependent variable, X is a known n × p matrix of explanatory variables, $\beta$ is a p × 1 vector of unknown regression parameters, and $\varepsilon$ is an n × 1 vector of disturbances with a zero mean and the variance–covariance matrix . is an identity matrix with the order n × n. The ordinary least-squares estimator (OLS) for $\beta$ in Equation (1) is defined as

$$\hat{\beta}_{ols} = J^{-1}X'y \qquad (2)$$

Where

Although the Ordinary Least Squares estimator is unbiased and efficient under classical assumptions, its performance deteriorates in the presence of multicollinearity or outliers. To address outliers in the response variable, robust methods such as Huber's M-estimator are commonly employed. In contrast, when multicollinearity is the primary concern, shrinkage techniques such as the Ordinary Ridge Regression (ORR) estimator are widely used to stabilise parameter estimates and is expressed as.

$$\hat{\beta}(k) = L\hat{\beta}, k \geq 0 \qquad (3)$$

Where where k denotes the biasing parameter (Hoerl, 1970). Since the Ordinary Ridge Regression (ORR) estimator remains sensitive to outliers in the response variable, Silvapulle (1991) proposed the Ordinary Ridge Regression M-estimator (ORRM) to enhance robustness against such contamination. The ORRM is expressed as:

$$\hat{\beta}_M(k) = L\hat{\beta}_M, k \geq 0 \qquad (4)$$

Where denotes the M – estimator and its obtained by minimizing the sum of scaled residuals as follows:

$$\hat{\beta}_M = \min_{\beta} \sum_{j=1}^{n} \varphi\left(\frac{\varepsilon_j}{v}\right) \qquad (5)$$

Where represents the residual and is a chosen robust objective function and v is a robust estimated of scale parameter. The estimator is derived by solving the corresponding M-estimation equations, and $\rho = \varphi'$ is a suitable selected function. In practice, the solution is computed using the iterative reweighted least squares (IRLS) algorithm, in which residual-based weights are employed to reduce the influence of outliers in the response direction (Huber, 1981; Hampel, 1986; Montgomery, 2001; Awaad, 2022).

Liu (1993) introduced an alternative shrinkage approach to mitigate multicollinearity by integrating features of the Stein (1956) estimator with the Ordinary Ridge Regression estimator, leading to what is now referred to as the Liu estimator:

$$\hat{\beta}(d) = N\hat{\beta}, d \geq 0 \qquad (6)$$

Where and represents the Liu biasing parameter. The estimator has been widely applied and further examined in subsequent studies (Akdeniz and Kaciranlar, 1995, Oladapo et al, 2023, Idowu, et al, 2023 and Owolabi et al, 2023). Despite its effectiveness in handling multicollinearity, the Liu estimator remains sensitive to contaminations of observations in the response variable. To enhance robustness, Arslan and Billor (2000) extended this approach by proposing the Liu M-estimator:

$$\hat{\beta}_M(d) = N\hat{\beta}_M, d \geq 0 \qquad (7)$$

Other biased estimators have been proposed to handle the problem of multicollinearity in regression model.

Kibra-Lukman (2020) proposed another one biasing parameter estimator aside the Ridge and Liu estimator to tackle the effect of multicollinearity known as KL estimator and the estimator is defined as:

$$\hat{\beta}(kl) = O\hat{\beta}, k \geq 0 \qquad (8)$$

Where and k is also the biasing parameter for the Kibra – Lukman estimator. Just as Liu estimator many studies also used the Kibra – Lukman estimator too. However, the KL estimator is affected by outliers in the y-direction. To address this, Majid et al (2022) proposed the KL- M estimator as:

$$\hat{\beta}_M(kl) = O\hat{\beta}_M, k \geq 0 \qquad (9)$$

From various studies and research, it has been established most of the times that two parameter estimator a more effective in handling the problem of multicollinearity that is encountered in econometric data set, in 2023, Badawaire et al (2023) proposed a two parameter estimator to tackle the problem of multicollinearity and the estimator is defined as:

$$\hat{\beta}(prop1) = NO\hat{\beta}, k \geq 0 \text{ and } 0 < d < 1 \qquad (10)$$

### Theoretical methodology of the proposed robust estimator

Given the presence of extreme observations that may adversely influence the proposed two-parameter estimator, a robust modification is introduced to enhance its resistance to outliers. The robust form of the new two-parameter estimator is therefore defined as follows:

$$\hat{\beta}_M(prop1) = NO\hat{\beta}_M, k \geq 0 \text{ and } 0 < d < 1 \qquad (12)$$

**Properties of the proposed estimator**

**Canonical form**

Consider the linear regression model expressed in its canonical form as:

$$y = Z\alpha + \varepsilon \qquad (13)$$

Where . Let H be an orthogonal matrix such that . Then the above estimate follows:

$$\hat{\alpha} = B^{-1}Z'y \qquad (14)$$

$$\hat{\alpha}_M = \min_{\alpha} \sum_{j=1}^{n} \varphi\left(\frac{y_j - z_j'\alpha}{v}\right) \qquad (15)$$

$$\hat{\alpha}_M(k) = [I_p + kB^{-1}]^{-1}\hat{\alpha}_M \qquad (16)$$

$$\hat{\alpha}_M(d) = [B + I_p]^{-1}[B + dI_p]\hat{\alpha}_M \qquad (17)$$

$$\hat{\alpha}_M(kl) = [B + kI_p]^{-1}[B - kI_p]\hat{\alpha}_M \qquad (18)$$

The proposed prop1 estimator of  is given by

$$\hat{\alpha}_M(prop1) = [B + I_p]^{-1}[B + dI_p][B + kI_p]^{-1}[B - kI_p]\hat{\alpha}_M \qquad (19)$$

The mean square error (MSE) of any estimator  is defined as

$$MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^T(\hat{\theta} - \theta) \qquad (20)$$

$$= tr(\mathrm{cov}(\hat{\theta})) + bias(\hat{\theta})^T\, bias(\hat{\theta}) \qquad (21)$$

The MSE of robust regression estimator is expressed below as

$$MSE(\hat{\alpha}_M) = \sum_{i=1}^{p} \Omega_{ii} \qquad (22)$$

where  is the diagonal element for  which is equivalent to  which is finite.

MSE for Robust Ridge estimator  is defined as:

$$MSE(\hat{\alpha}_M(k)) = \sum_{i=1}^{p} \frac{\lambda_i^2\Omega_{ii} + k^2\alpha_i^2}{(\lambda_i + k)^2} \qquad (23)$$

Likewise, the MSE for robust Liu estimator  denoted as  is expressed as:

$$MSE(\hat{\alpha}_M(d)) = \sum_{i=1}^{p} \frac{(\lambda_i + d)^2\Omega_{ii} + (1-d)^2\alpha_i^2}{(\lambda_i + 1)^2} \qquad (24)$$

The Robust Kibria and Lukman defined the canonical form of their MSE as follows:

$$MSE(\hat{\alpha}_M(kl)) = \sum_{i=1}^{p} \frac{(\lambda_i - k)^2\Omega_{ii}}{\lambda_i(\lambda_i + k)^2} + \sum_{i=1}^{p} \frac{4k^2\alpha_i^2}{(\lambda_i + k)^2} \qquad (25)$$

Following the works of by Badawaire et al (2023), the MSE of the robust version of the estimator (RPROP1) can be expressed as:

$$MSE(\hat{\alpha}_M(\Pr op1)) = \sum_{i=1}^{p}\left[\frac{(\lambda_i - k)^2(\lambda_i + d)^2\Omega_i}{\lambda_i(\lambda_i + I)^2(\lambda_i + k)^2}\right] + \sum_{i=1}^{p}\left[\frac{((2k+1-d)\lambda_i) + k(d+1)^2}{(\lambda_i + I)^2(\lambda_i + k)^2}\right]\hat{\alpha}_M \qquad (26)$$

**Determination of the biasing parameters**

Assume that $\hat{\alpha}_M \sim N(0,1)$. This implies that $\hat{\alpha}_M$ follows a normal distribution with mean zero, variance equal one and covariance matrix equals $B^2\Gamma^{-1}$.

When $\sqrt{n}(\hat{\theta}_M^2 - \theta) \sim N(0, B^2\Gamma^{-1})$, hence the assumption sustains most importantly for practical use, where

$$B^2 = \frac{c_0^2 E[\omega^2(\varepsilon / c_0)]}{E[\omega'(\varepsilon / c_0)]^2}.$$

Such that $c_0$ is the scale estimate.

Also, the unbiased estimator $\alpha_M = \hat{\alpha}_M$ that is $E(\alpha_M) = \hat{\alpha}_M$ and the unbiased estimator $\Omega_{ii} = B^2 / \Gamma_i$ such that .

$$B^2 = \frac{c^2(n-p)^{-1}\sum_{i=1}^{n}[\omega(\varepsilon_i / c)]^2}{\sum_{i=1}^{n}\left[\frac{1}{n}\omega'(\varepsilon_i / c)\right]^2}.$$

Biasing parameters for some robust estimators are hereby expressed as follows:

(i) Robust Ridge Regression (RRR) estimator biasing parameter k is defined as:

$$\hat{k} = \frac{pB^2}{\sum_{i=1}^{p}\hat{\alpha}_M}. \qquad (27)$$

(ii) Biasing parameter for Robust Liu estimator is defined as:

$$\hat{d} = 1 - B^2\left[\frac{\sum_{i=1}^{p}\frac{1}{\lambda_i(\lambda_i + 1)}}{\sum_{i=1}^{p}\frac{\hat{\alpha}_M}{(\lambda_i + 1)^2}}\right]. \qquad (28)$$

(iii) Biasing parameters for Robust Ridge-Liu estimator can be expressed as:

$$\hat{k} = \frac{1}{p}\sum_{i=1}^{p}\frac{B^2}{\hat{\alpha}_M^2 - d\left(\frac{B^2}{\lambda_i + \hat{\alpha}_M^2}\right)} \quad \text{and} \qquad (29)$$

$$\hat{d} = \min\left(\frac{\hat{\alpha}_M^2}{\frac{B^2}{\lambda_i} + \hat{\alpha}_M^2}\right). \qquad (30)$$

Since, the MSE of the robust version of the estimator (RPROP1) from equation (  ) is already expressed as below.

$$MSE(\hat{\alpha}_M(\Pr op1)) = \sum_{i=1}^{p}\left[\frac{(\lambda_i - k)^2(\lambda_i + d)^2\Omega_i}{\lambda_i(\lambda_i + I)^2(\lambda_i + k)^2}\right] + \sum_{i=1}^{p}\left[\frac{((2k+1-d)\lambda_i) + k(d+1)^2}{(\lambda_i + I)^2(\lambda_i + k)^2}\right]\hat{\alpha}_M$$

Considering d to be fixed, an optimal value of k is the value that minimizes . $MSE(\hat{\alpha}_M(\Pr op1))$.

Then, by differentiating w.r.t. k and equating to 0, the biasing k parameter is:

$$k_M(\Pr op1) = \frac{B^2\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\alpha_M^2}{B^2(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\alpha_M^2} \qquad (31)$$

For the sake of practicality, the estimated generalized biasing parameter will be expressed below as:

$$\hat{k}_M(\Pr op1) = \frac{\hat{B}^2\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\hat{\alpha}_M^2}{\hat{B}^2(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\hat{\alpha}_M^2} \qquad (32)$$

Equation (32) returns the biasing parameter for the RKL estimator when d =1, which is defined as follows:

$$\hat{k}(kl) = \frac{\hat{B}^2\lambda_i}{\hat{B}^2 + 2\lambda_i\hat{\alpha}_M^2} \qquad (33)$$

Equation (33) is said to be the generalized biasing parameter k of the PROP1 estimator, Oladapo et al (2023, 2024) suggested some versions of k. Hence, applying their concepts, the below shrinkage parameters are examined for the proposed estimator and they are defined as follows:

$$\hat{k}_{MED}(\Pr op1) = MEDIAN\left(\frac{\hat{B}^2\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\hat{\alpha}_M^2}{\hat{B}^2(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\hat{\alpha}_M^2}\right) \qquad (34)$$

$$\hat{k}_{AM}(\Pr op1) = MEAN\left(\frac{\hat{B}^2\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\hat{\alpha}_M^2}{\hat{B}^2(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\hat{\alpha}_M^2}\right) \qquad (35)$$

Considering k to be fixed, an optimal value of d is the value that minimizes .

Then, by differentiating w.r.t. d and equating to 0, we have :

$$d(\Pr op1) = \frac{\lambda_i(\lambda_i(2k+1)+k)\alpha_M^2 - B^2(\lambda_i - k)\lambda_i}{B^2(\lambda_i - k) + \lambda_i(k - \lambda_i)\alpha_M^2} \quad (36)$$

For the sake of practicality, the estimated generalized biasing parameter will be expressed below as:

$$\hat{d}(\Pr op1) = \frac{\lambda_i(\lambda_i(2\hat{k}+1)+\hat{k})\hat{\alpha}_M^2 - \hat{B}^2(\lambda_i - \hat{k})\lambda_i}{\hat{B}^2(\lambda_i - \hat{k}) + \lambda_i(\hat{k} - \lambda_i)\hat{\alpha}_M^2} \quad (37)$$

**Algorithm for the Simulation Design**

(i) Select the sample size say n.

(ii) Generate response variable using:

$$y = \beta_0 + \beta_1 x_2 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_p x_{ip} + \varepsilon_i \, , i = 1, 2, 3, \ldots, p.$$

(iii) Choose a percentage of the data to be replaced with outliers.

(iv) Choose a particular magnitude of outlier to invoke into the data to be randomly generated from step (ii)

(v) Randomly select those observations making up the percentage of the generated data to be replaced with outliers.

(vi) Invoke outliers into the data using; Y(i)outlier = f*Max (Yi) + Yi where *f* is the magnitude of outliers in the x-direction

(vii) Replace the outliers in original data generated in (iii)

(viii) Select Correlation levels say rho

(ix) Choose intercept

(x) Generate exogenous variables using equation:

$$x_{ij} = (1 - rho^2)^{\frac{1}{2}} z_{ij} + rhoz_{ip+1} \, i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, p$$

(xi) Obtain the MSE of the estimators of the model

(xii) Compute for each replicate the estimated MSE for each of the estimators by dividing the result in step (ix) by the number of replications.

The simulated MSE (SMSE) was used as the judgment criterion and is given by:

$$MSE(\beta^*) = \frac{1}{1000} \sum_{i=1}^{1000} (\beta^* - \beta)^2 \quad (38)$$

where $\beta^*$ is any of the estimators of both existing and proposed estimators.

## RESULTS AND DISCUSSION

### Simulation results

Tables 1–6 report the minimum mean squared error (MSE) values for the competing estimators under different combinations of sample size (n=20,40 and 100), error variance (sigma = 1, 3 and 5), correlation level (ϱ\rho = 0.80, 0.90, 0.95 and 0.99), number of explanatory variables (P=5 and 7), and outlier magnitude (my = 0,5,10) .The proposed robust estimators (Rprop1ME and Rprop1AM) are compared with existing estimators, namely the robust regression estimator (M), robust ridge (RRIDGE), robust Liu (RLIU), and robust Kibria–Lukman (RK–L).

**Effect of correlation (ϱ\rho)**

For P = 5, (Tables 1–3), Rprop1ME frequently attains the smallest MSE as rho increases from 0.80 to 0.95, particularly for n = 20 and 40. Under extreme collinearity (rho = 0.99), Rprop1ME maintains competitive performance and, in many cases, clearly outperforms RRIDGE, RLIU and RK–L. Rprop1AM also demonstrates substantial improvement over the existing estimators for moderate correlation levels; however, its performance becomes less stable in a few extreme scenarios (e.g., small n and very high rho), where isolated large MSE values appear.

For P = 7 (Tables 4–6), where dimensionality is increased, the superiority of the proposed estimators becomes even more evident. Both Rprop1ME and Rprop1AM produce substantially lower MSE values compared with the competing methods across most parameter settings, particularly when rho 0.90.

**Effect of sample size (n)**

As expected, increasing the sample size from n = 20 to 100 leads to a reduction in MSE for all estimators. Nevertheless, the relative advantage of Rprop1ME remains consistent. Even at n =100, where all estimators improve, Rprop1ME typically retains the smallest or near-smallest MSE, confirming that its gain is not limited to small samples.

**Effect of error variance (σ\sigma)**

An increase in sigma from 1 to 5 results in higher MSE values across all estimators. However, the growth rate is substantially slower for Rprop1ME and, in most cases, for Rprop1AM. This indicates improved stability under increased noise levels compared with the other existing robust estimators.

**Effect of outlier magnitude (my)**

Comparing Tables 1–3 (P = 5) and Tables 4–6 (P = 7) across my = 0, 5 and 10, the proposed estimators remain relatively stable as outlier magnitude increases. While the MSE of conventional robust estimators increases noticeably with increasing contamination magnitude, Rprop1ME, in particular, exhibits only moderate variation, demonstrating strong resistance to response-direction contamination.

**General performance pattern**

Across all configurations, the classical robust estimator (M) consistently records the largest MSE values, particularly as multicollinearity intensifies (ϱ→0.99) and as sigma increases. This confirms that robustness alone is insufficient in the presence of strong collinearity. Similarly, although RRIDGE, RLIU and RK–L improve substantially over M, their MSE values increase sharply when both the correlation level and variance are high. In contrast, the proposed estimators Rprop1ME and Rprop1AM generally yield markedly smaller MSE values under moderate to severe multicollinearity. The reduction is especially pronounced when n = 20 and 40, indicating improved small- and moderate-sample performance.

**Table 1**: Minimum MSE value when P=5 my =0 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.80 | 4.0050 | 0.8697 | 0.5343 | 0.7448 | 0.1772 | 0.1640 |
| | | 0.90 | 5.2304 | 1.7471 | 1.0060 | 1.4373 | 0.6357 | 0.4359 |
| | | 0.95 | 7.8958 | 3.5523 | 2.0011 | 2.8809 | 1.0644 | 1.6214 |
| | | 0.99 | 30.5799 | 18.5538 | 10.5431 | 15.0694 | 10.2249 | 60.4906 |
| 3 | | 0.80 | 12.5631 | 9.2307 | 7.3947 | 8.8796 | 2.0298 | 1.5107 |
| | | 0.90 | 23.8935 | 19.9537 | 16.2259 | 19.1055 | 2.2064 | 2.3268 |
| | | 0.95 | 48.0781 | 42.7312 | 34.9530 | 40.8330 | 5.7978 | 11.0699 |
| | | 0.99 | 252.6837 | 234.9474 | 192.8721 | 224.1839 | 116.9866 | 158.8171 |
| 5 | | 0.80 | 29.7151 | 26.3617 | 23.1733 | 25.9295 | 7.1187 | 5.0096 |
| | | 0.90 | 61.2772 | 57.2854 | 50.7798 | 56.2748 | 6.6253 | 5.8369 |
| | | 0.95 | 128.5301 | 123.0560 | 109.3967 | 120.8049 | 15.4644 | 27.4949 |
| | | 0.99 | 697.1037 | 678.4838 | 604.1569 | 665.7361 | 358.6171 | 396.5786 |
| 1 | 40 | 0.80 | 3.2535 | 0.3384 | 0.2342 | 0.2990 | 0.0930 | 0.2150 |
| | | 0.90 | 3.6162 | 0.5953 | 0.3691 | 0.5047 | 0.1448 | 0.3447 |
| | | 0.95 | 4.3295 | 1.0843 | 0.6089 | 0.8938 | 0.3486 | 0.2067 |
| | | 0.99 | 10.0565 | 4.8126 | 2.4997 | 3.9107 | 2.1329 | 3.2671 |
| 3 | | 0.80 | 6.3850 | 3.3692 | 2.5547 | 3.2544 | 1.2748 | 1.6432 |
| | | 0.90 | 9.5275 | 6.3106 | 4.8085 | 6.0571 | 1.6007 | 1.4767 |
| | | 0.95 | 15.8930 | 12.2749 | 9.3649 | 11.7349 | 1.9194 | 1.2723 |
| | | 0.99 | 67.4239 | 60.5104 | 46.7122 | 57.6545 | 10.4709 | 21.1033 |
| 5 | | 0.80 | 12.6472 | 9.5961 | 8.0134 | 9.4413 | 4.6648 | 4.3308 |
| | | 0.90 | 21.3516 | 18.0807 | 15.1913 | 17.7567 | 5.6705 | 4.2230 |
| | | 0.95 | 39.0246 | 35.3219 | 29.7621 | 34.6585 | 5.8015 | 3.7162 |
| | | 0.99 | 182.1775 | 174.9593 | 148.9049 | 171.5309 | 32.9875 | 54.3692 |
| 1 | 100 | 0.80 | 3.0567 | 0.1561 | 0.1003 | 0.1397 | 0.0686 | 0.1283 |
| | | 0.90 | 3.2325 | 0.2908 | 0.1772 | 0.2480 | 0.0880 | 0.1859 |
| | | 0.95 | 1.7980 | 0.5398 | 0.3249 | 0.4414 | 0.2057 | 0.4718 |
| | | 0.99 | 6.4407 | 2.3443 | 1.3435 | 1.8826 | 2.7387 | 0.5353 |
| 3 | | 0.80 | 4.4401 | 1.4850 | 1.1555 | 1.4362 | 0.7093 | 1.2180 |
| | | 0.90 | 5.9952 | 2.9351 | 2.3384 | 2.8119 | 1.0848 | 1.7801 |
| | | 0.95 | 7.3313 | 5.8652 | 4.7480 | 5.5871 | 1.4411 | 1.4899 |
| | | 0.99 | 34.7194 | 29.5119 | 24.3498 | 27.9972 | 3.7982 | 7.5438 |
| 5 | | 0.80 | 7.1970 | 4.2096 | 3.6205 | 4.1404 | 2.5754 | 2.9074 |
| | | 0.90 | 11.5082 | 8.3956 | 7.3805 | 8.2334 | 3.8142 | 3.8271 |
| | | 0.95 | 18.4153 | 16.8981 | 15.0222 | 16.5469 | 4.8531 | 3.8636 |
| | | 0.99 | 91.2419 | 85.7609 | 76.7865 | 83.8966 | 11.8846 | 17.9811 |

P=Number of explanatory variable My=magnitude of outlier Py=percentage of outlier

**Table 2**: Minimum MSE value when P=5  my =5 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.80 | 4.0674 | 0.9203 | 0.5582 | 0.7901 | 0.2061 | 0.1549 |
| | | 0.90 | 5.3692 | 1.8587 | 1.0604 | 1.5319 | 0.6596 | 0.4981 |
| | | 0.95 | 8.1932 | 3.7932 | 2.1404 | 3.0785 | 1.7827 | 0.9646 |
| | | 0.99 | 32.1864 | 19.8764 | 11.3470 | 16.1343 | 14.3646 | 50.8788 |
| 3 | | 0.80 | 13.9222 | 10.5722 | 8.6552 | 10.2104 | 2.4413 | 1.7873 |
| | | 0.90 | 26.3727 | 22.3920 | 18.5085 | 21.5204 | 2.4807 | 2.7926 |
| | | 0.95 | 52.7256 | 47.3217 | 39.2575 | 45.3793 | 6.0794 | 12.2307 |
| | | 0.99 | 278.6312 | 260.6461 | 217.4316 | 249.6364 | 124.4003 | 183.3321 |
| 5 | | 0.80 | 34.2365 | 30.8541 | 27.5820 | 30.4125 | 8.7312 | 6.2680 |
| | | 0.90 | 70.6829 | 66.6627 | 60.0273 | 65.6298 | 7.7110 | 7.0839 |
| | | 0.95 | 147.3611 | 141.8766 | 128.0161 | 139.5875 | 17.7530 | 30.4429 |
| | | 0.99 | 796.3131 | 777.6412 | 702.3913 | 764.6792 | 409.1909 | 505.2678 |
| 1 | 40 | 0.80 | 3.2815 | 0.3658 | 0.2526 | 0.3250 | 0.1008 | 0.2500 |
| | | 0.90 | 3.6717 | 0.6467 | 0.3989 | 0.5530 | 0.1411 | 0.2597 |
| | | 0.95 | 4.4418 | 1.1849 | 0.6712 | 0.9856 | 0.4502 | 0.2216 |
| | | 0.99 | 10.6360 | 5.3205 | 2.8095 | 4.3569 | 4.2586 | 3.6278 |
| 3 | | 0.80 | 6.6484 | 3.6361 | 2.7711 | 3.5190 | 1.3940 | 1.9600 |
| | | 0.90 | 10.0399 | 6.8251 | 5.2331 | 6.5670 | 1.7514 | 1.4664 |
| | | 0.95 | 16.9186 | 13.3000 | 10.2312 | 12.7531 | 2.1063 | 1.3738 |
| | | 0.99 | 72.6603 | 65.7264 | 51.1335 | 62.7995 | 11.5721 | 22.6172 |
| 5 | | 0.80 | 13.3870 | 10.3448 | 8.6597 | 10.1890 | 5.0376 | 4.9340 |
| | | 0.90 | 22.7820 | 19.5201 | 16.4738 | 19.1956 | 6.1849 | 4.5496 |
| | | 0.95 | 41.8818 | 38.1873 | 32.3604 | 37.5193 | 6.3404 | 3.9563 |
| | | 0.99 | 196.7575 | 189.5391 | 162.2004 | 186.0511 | 35.7183 | 59.2471 |
| 1 | 100 | 0.80 | 3.0678 | 0.1700 | 0.1108 | 0.1530 | 0.0748 | 0.1379 |
| | | 0.90 | 3.2580 | 0.3183 | 0.1969 | 0.2736 | 0.1023 | 0.2933 |
| | | 0.95 | 1.8680 | 0.5944 | 0.3530 | 0.4912 | 0.2764 | 0.4670 |
| | | 0.99 | 6.7533 | 2.6201 | 1.5264 | 2.1227 | 1.5720 | 0.5848 |
| 3 | | 0.80 | 4.5630 | 1.6179 | 1.2744 | 1.5680 | 0.7979 | 1.2493 |
| | | 0.90 | 6.2547 | 3.2065 | 2.5935 | 3.0797 | 1.2134 | 1.9311 |
| | | 0.95 | 7.9179 | 6.4229 | 5.2560 | 6.1382 | 1.6646 | 1.5888 |
| | | 0.99 | 37.6234 | 32.4278 | 26.9045 | 30.8594 | 4.2955 | 7.3918 |
| 5 | | 0.80 | 7.5514 | 4.5824 | 3.9814 | 4.5121 | 2.8258 | 3.2748 |
| | | 0.90 | 12.2463 | 9.1574 | 8.0995 | 8.9944 | 4.1937 | 4.4232 |
| | | 0.95 | 20.0205 | 18.4632 | 16.4763 | 18.1087 | 5.5041 | 4.2297 |
| | | 0.99 | 99.3591 | 93.9325 | 84.3685 | 92.0219 | 13.3210 | 19.4519 |

**Table 3**: Minimum MSE value when P=5 my =10 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.80 | 4.0675 | 0.9204 | 0.5582 | 0.7901 | 0.2061 | 0.1549 |
| | | 0.90 | 5.3694 | 1.8588 | 1.0605 | 1.5320 | 0.6595 | 0.4942 |
| | | 0.95 | 8.1935 | 3.7934 | 2.1406 | 3.0787 | 1.7829 | 0.9644 |
| | | 0.99 | 32.1880 | 19.8779 | 11.3480 | 16.1356 | 14.3644 | 50.8150 |
| 3 | | 0.80 | 13.1127 | 9.7622 | 7.8788 | 9.4028 | 2.1303 | 1.5407 |
| | | 0.90 | 25.1202 | 21.1479 | 17.3153 | 20.2800 | 2.1712 | 2.5410 |
| | | 0.95 | 50.7313 | 45.3304 | 37.3537 | 43.3942 | 5.7775 | 11.8774 |
| | | 0.99 | 267.2319 | 249.3126 | 206.3042 | 238.3356 | 120.0036 | 177.4486 |
| 5 | | 0.80 | 31.2415 | 27.8667 | 24.6666 | 27.4272 | 7.5836 | 5.3225 |
| | | 0.90 | 64.6856 | 60.6474 | 54.1325 | 59.6205 | 6.5701 | 6.1173 |
| | | 0.95 | 135.8462 | 130.3035 | 116.6639 | 128.0263 | 16.2056 | 28.3985 |
| | | 0.99 | 737.2144 | 718.4008 | 644.3719 | 705.5083 | 380.8570 | 476.7965 |
| 1 | 40 | 0.80 | 3.2815 | 0.3658 | 0.2526 | 0.3250 | 0.1008 | 0.2500 |
| | | 0.90 | 3.6717 | 0.6467 | 0.3989 | 0.5530 | 0.1412 | 0.2595 |
| | | 0.95 | 4.4419 | 1.1850 | 0.6713 | 0.9857 | 0.4507 | 0.2216 |
| | | 0.99 | 10.6364 | 5.3210 | 2.8097 | 4.3573 | 4.2694 | 3.6395 |
| 3 | | 0.80 | 6.6484 | 3.6361 | 2.7710 | 3.5190 | 1.3941 | 1.9574 |
| | | 0.90 | 10.0396 | 6.8247 | 5.2328 | 6.5666 | 1.7515 | 1.4664 |
| | | 0.95 | 16.9178 | 13.2991 | 10.2304 | 12.7522 | 2.1039 | 1.3739 |
| | | 0.99 | 72.6594 | 65.7255 | 51.1331 | 62.7986 | 11.5688 | 22.6425 |
| 5 | | 0.80 | 13.3851 | 10.3427 | 8.6574 | 10.1869 | 5.0349 | 4.9319 |
| | | 0.90 | 22.7808 | 19.5184 | 16.4734 | 19.1939 | 6.1841 | 4.5488 |
| | | 0.95 | 41.8788 | 38.1838 | 32.3610 | 37.5159 | 6.3393 | 3.9557 |
| | | 0.99 | 196.7303 | 189.5124 | 162.1965 | 186.0251 | 35.7092 | 59.2722 |
| 1 | 100 | 0.80 | 3.0678 | 0.1700 | 0.1108 | 0.1530 | 0.0748 | 0.1379 |
| | | 0.90 | 3.2580 | 0.3183 | 0.1969 | 0.2737 | 0.1024 | 0.2925 |
| | | 0.95 | 1.8680 | 0.5945 | 0.3530 | 0.4912 | 0.2767 | 0.4670 |
| | | 0.99 | 6.7536 | 2.6204 | 1.5266 | 2.1230 | 1.5615 | 0.5848 |
| 3 | | 0.80 | 4.5631 | 1.6179 | 1.2745 | 1.5681 | 0.7978 | 1.2489 |
| | | 0.90 | 6.2548 | 3.2066 | 2.5936 | 3.0798 | 1.2134 | 1.9319 |
| | | 0.95 | 7.9181 | 6.4231 | 5.2563 | 6.1384 | 1.6644 | 1.5885 |
| | | 0.99 | 37.6246 | 32.4290 | 26.9056 | 30.8606 | 4.2944 | 7.3918 |
| 5 | | 0.80 | 7.5514 | 4.5825 | 3.9815 | 4.5121 | 2.8259 | 3.2751 |
| | | 0.90 | 12.2465 | 9.1575 | 8.0996 | 8.9946 | 4.1938 | 4.4230 |
| | | 0.95 | 20.0209 | 18.4636 | 16.4766 | 18.1091 | 5.5123 | 4.2336 |
| | | 0.99 | 99.3609 | 93.9342 | 84.3704 | 92.0237 | 13.3322 | 19.4475 |

**Table 4**: Minimum MSE value when P=7, my =0 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.8 | 6.4588 | 2.7977 | 1.5574 | 2.2796 | 1.4649 | 0.5765 |
| | | 0.9 | 10.7801 | 5.8426 | 3.2815 | 4.8574 | 3.0204 | 0.9514 |
| | | 0.95 | 19.5235 | 11.9486 | 6.7740 | 10.0867 | 4.6366 | 4.7386 |
| | | 0.99 | 89.8835 | 60.9266 | 35.0616 | 52.1786 | 26.5352 | 247.6826 |
| 3 | | 0.8 | 35.9353 | 31.6486 | 25.0076 | 30.2495 | 5.0013 | 5.9157 |
| | | 0.9 | 75.0203 | 68.6654 | 55.1076 | 65.6929 | 10.1168 | 19.2735 |
| | | 0.95 | 153.8448 | 143.2578 | 115.8519 | 137.1358 | 26.3956 | 43.3970 |
| | | 0.99 | 787.4151 | 742.5772 | 604.8231 | 711.1979 | 307.5996 | 263.6052 |
| 5 | | 0.8 | 94.9290 | 90.5559 | 78.4687 | 88.8998 | 15.0047 | 16.7453 |
| | | 0.9 | 203.5582 | 196.9925 | 172.4054 | 193.4448 | 37.6870 | 57.1130 |
| | | 0.95 | 422.5676 | 411.5105 | 361.9006 | 404.1551 | 110.6547 | 145.1251 |
| | | 0.99 | 2182.6527 | 2135.1743 | 1886.1696 | 2097.1798 | 1177.3397 | 761.3838 |
| 1 | 40 | 0.8 | 2.0418 | 0.6169 | 0.3917 | 0.5131 | 0.2251 | 0.3909 |
| | | 0.9 | 4.2190 | 1.1650 | 0.6682 | 0.9379 | 0.6332 | 0.5544 |
| | | 0.95 | 5.7760 | 2.2443 | 1.2444 | 1.8161 | 1.5143 | 0.5484 |
| | | 0.99 | 17.0162 | 10.8489 | 6.0065 | 9.0618 | 3.8328 | 5.0762 |
| 3 | | 0.8 | 7.9765 | 6.3604 | 4.8733 | 6.0897 | 1.8745 | 2.0943 |
| | | 0.9 | 16.1729 | 12.8255 | 10.0670 | 12.2436 | 2.5456 | 1.9958 |
| | | 0.95 | 30.1504 | 26.0279 | 20.7045 | 24.8404 | 3.6187 | 5.1480 |
| | | 0.99 | 142.8223 | 133.5280 | 107.5427 | 127.4785 | 22.3592 | 47.0722 |
| 5 | | 0.8 | 19.8140 | 18.1246 | 15.3941 | 17.7886 | 6.7150 | 5.9593 |
| | | 0.9 | 40.1136 | 36.7282 | 31.7435 | 36.0291 | 8.2347 | 5.9953 |
| | | 0.95 | 78.9468 | 74.7489 | 65.1804 | 73.3290 | 11.3529 | 13.6398 |
| | | 0.99 | 394.3157 | 384.3894 | 337.6471 | 377.1461 | 99.0987 | 149.3773 |
| 1 | 100 | 0.8 | 3.0517 | 0.2926 | 0.1774 | 0.2481 | 0.1007 | 0.2990 |
| | | 0.9 | 3.4352 | 0.5766 | 0.3397 | 0.4634 | 0.3139 | 0.6495 |
| | | 0.95 | 4.2030 | 1.1167 | 0.6222 | 0.8897 | 0.7022 | 0.8266 |
| | | 0.99 | 10.4221 | 5.2787 | 2.7971 | 4.3699 | 2.9159 | 0.8740 |
| 3 | | 0.8 | 5.6777 | 2.8611 | 2.2345 | 2.7439 | 1.2070 | 2.0010 |
| | | 0.9 | 9.0574 | 6.0473 | 4.7565 | 5.7757 | 1.9427 | 2.1140 |
| | | 0.95 | 15.9471 | 12.5343 | 9.8999 | 11.9534 | 2.8420 | 2.2470 |
| | | 0.99 | 72.0192 | 65.2440 | 51.7462 | 62.1156 | 7.1584 | 17.8791 |
| 5 | | 0.8 | 10.9486 | 8.1238 | 6.9435 | 7.9796 | 4.1961 | 4.7991 |
| | | 0.9 | 20.3277 | 17.2961 | 14.9205 | 16.9687 | 6.2276 | 5.6759 |
| | | 0.95 | 39.4724 | 36.0166 | 31.2086 | 35.3128 | 8.0816 | 6.2688 |
| | | 0.99 | 195.2989 | 188.3017 | 163.7092 | 184.4950 | 32.8825 | 56.5583 |

**Table 5**: Minimum MSE value when P=7, my =5 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.8 | 6.3648 | 2.7189 | 1.4970 | 2.2006 | 1.2918 | 0.5776 |
| | | 0.9 | 10.5917 | 5.6738 | 3.1582 | 4.6922 | 2.4729 | 0.8834 |
| | | 0.95 | 19.1497 | 11.5976 | 6.5021 | 9.7406 | 4.0125 | 3.3924 |
| | | 0.99 | 88.0616 | 59.1065 | 33.6163 | 50.3727 | 25.0716 | 330.9469 |
| 3 | | 0.8 | 41.9815 | 37.6887 | 30.8515 | 36.2714 | 6.1640 | 7.3207 |
| | | 0.9 | 87.1943 | 80.8459 | 67.0260 | 77.8387 | 12.9119 | 22.2270 |
| | | 0.95 | 179.4936 | 168.9110 | 141.0617 | 162.7271 | 34.0486 | 49.5453 |
| | | 0.99 | 929.8582 | 885.0395 | 744.6242 | 853.3877 | 399.6563 | 325.8050 |
| 5 | | 0.8 | 118.2826 | 113.9559 | 101.5982 | 112.2926 | 20.9758 | 22.0780 |
| | | 0.9 | 256.1721 | 249.7506 | 224.2378 | 246.1878 | 49.4865 | 68.9163 |
| | | 0.95 | 535.5500 | 524.7125 | 473.4124 | 517.3252 | 144.3225 | 176.0811 |
| | | 0.99 | 2766.8819 | 2719.9550 | 2464.4563 | 2681.9581 | 1569.2422 | 1023.9602 |
| 1 | 40 | 0.8 | 2.1054 | 0.6722 | 0.4298 | 0.5638 | 0.2312 | 0.4283 |
| | | 0.9 | 4.3400 | 1.2745 | 0.7456 | 1.0360 | 0.8066 | 0.5902 |
| | | 0.95 | 6.0208 | 2.4641 | 1.3928 | 2.0113 | 1.2887 | 0.6023 |
| | | 0.99 | 18.2784 | 11.9641 | 6.7580 | 10.0411 | 3.9185 | 4.2026 |
| 3 | | 0.8 | 8.5345 | 6.9071 | 5.3435 | 6.6273 | 2.1276 | 2.6834 |
| | | 0.9 | 17.2719 | 13.9228 | 11.0233 | 13.3276 | 2.7928 | 2.2097 |
| | | 0.95 | 32.3685 | 28.2422 | 22.6564 | 27.0297 | 4.0719 | 5.5937 |
| | | 0.99 | 154.1250 | 144.7585 | 117.5014 | 138.5746 | 25.4224 | 52.7096 |
| 5 | | 0.8 | 21.3750 | 19.6722 | 16.8148 | 19.3274 | 7.2889 | 6.4287 |
| | | 0.9 | 43.2033 | 39.8225 | 34.6087 | 39.1142 | 8.8060 | 6.4534 |
| | | 0.95 | 85.1635 | 80.9732 | 70.9629 | 79.5373 | 12.1727 | 14.6903 |
| | | 0.99 | 425.8717 | 415.8909 | 367.1410 | 408.5524 | 110.0609 | 190.8273 |
| 1 | 100 | 0.8 | 3.0885 | 0.3260 | 0.2034 | 0.2786 | 0.1129 | 0.3071 |
| | | 0.9 | 3.5140 | 0.6461 | 0.3777 | 0.5258 | 0.3617 | 0.7743 |
| | | 0.95 | 4.3673 | 1.2592 | 0.7213 | 1.0172 | 0.8744 | 0.7718 |
| | | 0.99 | 11.2796 | 6.0174 | 3.2859 | 5.0295 | 2.7212 | 0.9522 |
| 3 | | 0.8 | 6.0067 | 3.1848 | 2.5124 | 3.0646 | 1.3367 | 2.3279 |
| | | 0.9 | 9.7642 | 6.7439 | 5.3533 | 6.4613 | 2.1208 | 2.7107 |
| | | 0.95 | 17.4229 | 13.9909 | 11.1858 | 13.3900 | 3.0598 | 2.6868 |
| | | 0.99 | 79.7306 | 72.8713 | 58.6472 | 69.6798 | 8.7179 | 20.7687 |
| 5 | | 0.8 | 11.8611 | 9.0312 | 7.7681 | 8.8826 | 4.6641 | 5.1811 |
| | | 0.9 | 22.2897 | 19.2490 | 16.7346 | 18.9141 | 6.7747 | 5.9898 |
| | | 0.95 | 43.5702 | 40.0980 | 35.0758 | 39.3831 | 9.2958 | 7.3500 |
| | | 0.99 | 216.7154 | 209.6495 | 184.2633 | 205.8249 | 38.3126 | 64.1252 |

**Table 6**: Minimum MSE value when P=7, my =10 and py=0.1

| sigma | n | rho | M | RRIDGE | RLIU | RK-L | Rprop1ME | Rprop1AM |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.8 | 6.3650 | 2.7191 | 1.4971 | 2.2008 | 1.2912 | 0.5778 |
| | | 0.9 | 10.5921 | 5.6741 | 3.1585 | 4.6925 | 2.4752 | 0.8835 |
| | | 0.95 | 19.1507 | 11.5985 | 6.5028 | 9.7414 | 3.9945 | 3.3949 |
| | | 0.99 | 88.0671 | 59.1112 | 33.6200 | 50.3770 | 25.1444 | 330.9323 |
| 3 | | 0.8 | 35.2767 | 31.0416 | 24.5539 | 29.6455 | 4.5639 | 5.9463 |
| | | 0.9 | 73.6221 | 67.3449 | 54.1211 | 64.3756 | 9.6112 | 19.1740 |
| | | 0.95 | 151.0344 | 140.5655 | 113.8399 | 134.4495 | 24.6079 | 41.4634 |
| | | 0.99 | 773.5609 | 729.0454 | 594.5631 | 697.6956 | 294.0543 | 260.2501 |
| 5 | | 0.8 | 92.9881 | 88.7097 | 76.9522 | 87.0622 | 14.4867 | 16.3683 |
| | | 0.9 | 199.3074 | 192.8700 | 169.0092 | 189.3316 | 36.4196 | 56.7004 |
| | | 0.95 | 413.8649 | 403.0007 | 354.8120 | 395.6691 | 107.2644 | 142.8213 |
| | | 0.99 | 2138.5077 | 2091.5487 | 1849.4060 | 2053.7551 | 1146.5563 | 726.9559 |
| 1 | 40 | 0.8 | 2.1054 | 0.6723 | 0.4298 | 0.5639 | 0.2312 | 0.4283 |
| | | 0.9 | 4.3401 | 1.2747 | 0.7458 | 1.0362 | 0.8070 | 0.5901 |
| | | 0.95 | 6.0212 | 2.4644 | 1.3929 | 2.0115 | 1.2836 | 0.6023 |
| | | 0.99 | 18.2802 | 11.9656 | 6.7590 | 10.0424 | 3.8988 | 4.2161 |
| 3 | | 0.8 | 8.5343 | 6.9068 | 5.3432 | 6.6271 | 2.1271 | 2.6827 |
| | | 0.9 | 17.2726 | 13.9235 | 11.0242 | 13.3283 | 2.7919 | 2.2098 |
| | | 0.95 | 32.3696 | 28.2433 | 22.6573 | 27.0308 | 4.0724 | 5.5936 |
| | | 0.99 | 154.1307 | 144.7642 | 117.5064 | 138.5802 | 25.4012 | 52.6689 |
| 5 | | 0.8 | 21.3554 | 19.6520 | 16.7935 | 19.3074 | 7.2803 | 6.4229 |
| | | 0.9 | 43.1740 | 39.7944 | 34.5816 | 39.0862 | 8.7960 | 6.4479 |
| | | 0.95 | 85.1196 | 80.9307 | 70.9246 | 79.4944 | 12.1617 | 14.6868 |
| | | 0.99 | 425.6974 | 415.7158 | 366.9917 | 408.3758 | 110.0247 | 190.6224 |
| 1 | 100 | 0.8 | 3.0885 | 0.3260 | 0.2034 | 0.2787 | 0.1129 | 0.3071 |
| | | 0.9 | 3.5141 | 0.6462 | 0.3777 | 0.5259 | 0.3619 | 0.7743 |
| | | 0.95 | 4.3674 | 1.2593 | 0.7214 | 1.0174 | 0.8712 | 0.7723 |
| | | 0.99 | 11.2802 | 6.0180 | 3.2864 | 5.0300 | 2.7144 | 0.9527 |
| 3 | | 0.8 | 6.0067 | 3.1849 | 2.5125 | 3.0646 | 1.3368 | 2.3277 |
| | | 0.9 | 9.7643 | 6.7440 | 5.3534 | 6.4614 | 2.1205 | 2.7105 |
| | | 0.95 | 17.4231 | 13.9911 | 11.1859 | 13.3902 | 3.0602 | 2.6865 |
| | | 0.99 | 79.7314 | 72.8722 | 58.6482 | 69.6807 | 8.7105 | 20.7760 |
| 5 | | 0.8 | 11.8612 | 9.0313 | 7.7683 | 8.8827 | 4.6643 | 5.1815 |
| | | 0.9 | 22.2900 | 19.2493 | 16.7349 | 18.9143 | 6.7749 | 5.9899 |
| | | 0.95 | 43.5706 | 40.0984 | 35.0763 | 39.3836 | 9.2984 | 7.3506 |
| | | 0.99 | 216.7183 | 209.6524 | 184.2659 | 205.8278 | 38.3053 | 64.1255 |

## Real Life Application

The proposed robust estimators were evaluated using the Hussain dataset (Eledum & Zahri, 2013), where the regression model relates the value of a manufactured product to imported intermediate, capital, and raw materials. Diagnostic analysis revealed severe multicollinearity, with VIFs of 173.67, 131.99, and 129.79, and a condition number of 7184.20, alongside approximately 10% outlier observations. The regression model is specified as:

$$y_i = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \quad (39)$$

Where $y_i$ represents the value of a manufactured product, $x_1$ denotes the value of imported intermediate commodities, $x_2$ represents the value of imported capital commodities, and $x_3$ denotes the value of imported raw materials. Table 7 shows that the proposed RProp1 estimator achieved the lowest mean squared error, confirming its superior performance and providing empirical support for the theoretical and simulation results.

**Table 7**: MSE and coefficients when Hussain dataset is used

| Estimator | k | d | X1 | X2 | X3 | Bias2 | Variance | MSE |
|---|---|---|---|---|---|---|---|---|
| M | | | -1.366 | 1.74131 | -1.8614 | 0 | 2.81452 | 2.81452 |
| RRidge | 0.1 | | -1.366 | 1.74131 | -1.8614 | 4.50E-10 | 2.81447 | 2.81447 |
| RLiu | | 0.1 | -1.366 | 1.74129 | -1.8613 | 3.64E-08 | 2.814 | 2.814 |
| RKL | 0.1 | | -1.366 | 1.7413 | -1.8614 | 1.80E-09 | 2.81441 | 2.81441 |
| Rprop1 | 0.1 | 0.1 | -1.366 | 1.74129 | -1.8612 | 5.44E-08 | **2.81388** | **2.81388** |

## CONCLUSION

This study proposed a new robust two-parameter shrinkage estimator for linear regression models affected simultaneously by multicollinearity and outliers. The Rprop1 estimator was developed within the canonical framework, and its statistical properties were established through mean squared error (MSE) analysis. Monte Carlo simulation results demonstrate that the proposed estimator, particularly Rprop1ME, delivers the most consistent and efficient performance across diverse experimental settings, including varying sample sizes, correlation levels, noise variances, dimensional structures, and outlier magnitudes. Rprop1AM also performs competitively, though with slight instability under extreme multicollinearity in small samples. Overall, the findings confirm that combining robustness with dual-parameter shrinkage substantially improves estimation stability and accuracy and it's also supported when applied to real dataset. The proposed estimators therefore provide practical and theoretically sound alternatives to existing robust and shrinkage methods when multicollinearity and outliers occur concurrently. Future studies may extend the approach to generalized linear and high-dimensional modeling frameworks.

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