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Modelling Regime-Specific Dependence Structure and Investment Risk Implications in Stock Markets using Copula-Switching GARCH-GED Models.

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ABSTRACT

A well-known traditional GARCH model assumes normal innovations which do not adequately capture sudden variations typically caused by economic shocks or disturbances. This has necessitated the need to develop non-linear, distributional and robust models. In this study, a new set of GARCH models with smooth transition non-linearities and novel innovation distributions are developed to improve the modeling and forecasting of stock returns volatilities in the Nigeria /US stock markets' Daily data on Heating Oil, Crude Oil, and Gasoline regular spot prices (Naira/US per Dollar) from 1985 to 2025 were obtained from the U.S. Energy Information Administration (EIA) website (https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm). This study was carried out using copula-based regime switching GARCH Generalized Error distribution (GED) model and a hidden Markov model. The copula switching GARCH (CoS GARCH) framework showed that the spot prices of crude oil, heating oil and gasoline demonstrated distinct patterns of volatility clustering and distributions with heavy and notable interdependence among different regimes. The estimated transition probability matrix indicated that the Markov chain associated with the volatility states displayed significant persistence. The equations for the conditional means indicated that returns were marginally different across the regimes, which aligns with the established observation that energy price returns have small means compared with their variances. The findings of the study therefore established the presence of heavy tails, clustering of volatility, structural changes, and significant interdependence in energy markets.

INTRODUCTION

Nigerian stock returns are widely recognized to exhibit high volatility, non-normal distributions, and pronounced nonlinear behavior, reflecting the market's sensitivity to macroeconomic shocks, policy changes, political instability, and global financial disturbances. Empirical evidence indicates that returns on the Nigerian Exchange Group (NGX) exhibit volatility clustering, leverage effects, skewness, and leptokurtosis, making them fundamentally different from the Gaussian assumptions typically used in financial modeling (Emenogu *et al.*, 2020). These characteristics underscore the need for more flexible and adaptive volatility models capable of capturing extreme movements and regime-dependent dynamics inherent in Nigerian financial markets.

Several empirical studies have attempted to model and forecast volatility in the Nigerian stock market using variants of the GARCH model. For instance, Adenomo *et al.* (2024) applied the EGARCH-X model with non-normal innovations and demonstrated the superiority of skewed Student's t distributions in capturing extreme return movements. Similarly, Emenogu *et al.* (2020) analyzed volatility and Value-at-Risk (VaR) using multiple GARCH models and found significant evidence of volatility persistence and leverage effects. Studies such as Abdulaziz *et al.* (2025) and Ogundeji *et al.* (2021) also highlight the importance of selecting appropriate innovation distributions particularly heavy-tailed and skewed forms to better reflect empirical return characteristics in Nigerian data.

Despite these efforts, substantial modeling gaps remain. Most existing studies rely on symmetric or linear GARCH variants that do not account for nonlinear transitions or regime switches, even though Nigerian market returns are characterized by abrupt shifts between restful and turbulent periods. Traditional GARCH models also struggle to capture smooth transitions between market conditions or accommodate time-varying dependence structures in multivariate settings. The few studies that incorporate nonlinearities such as STR or STAR-GARCH applications in other contexts have rarely been extended to African or Nigerian financial markets.

Moreover, the literature indicates persistent evidence of skewness and leptokurtosis in Nigerian stock returns, yet only a limited number of studies explicitly integrate skewed or heavy-tailed innovations into volatility models. This creates a methodological gap, particularly as extreme events, asymmetric effects, and fat tails are critical characteristics of financial risk modeling. Given these limitations, there is a growing need for more flexible and adaptive volatility models that incorporate smooth transitional mechanisms, hidden regimes, heavy-tailed innovations, and advanced dependence structures. This study addresses these gaps by developing two novel models the Bayesian Regime-Adaptive GARCH (BRAG-GARCH) and the Copula-Switching GARCH (CoS-GARCH) designed to better capture the nonlinear, asymmetric, and regime-dependent behavior of Nigerian

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stock return volatility. These models provide improved interpretability, greater flexibility in capturing structural breaks, and enhanced forecasting accuracy for risk management and financial decision-making.

Justification of the Study

Despite the extensive use of GARCH-family models in modeling Nigerian stock market volatility, existing studies largely rely on linear or symmetric specifications that fail to capture the complex empirical realities of Nigerian stock returns, such as nonlinear transitions, regime shifts, skewness, and heavy tails. Traditional GARCH models often assume normal innovations (Aako and Alabi, 2019; Usman *et al.*, 2017), and they do not adequately model abrupt changes caused by economic shocks, policy shifts, or market turbulence, leading to biased forecasts and poor risk estimation. Furthermore, advanced approaches such as Bayesian regime-switching frameworks, smooth-transition GARCH models, and copula-based dependence structures remain underexplored in the Nigerian context. This gap creates a critical need for more flexible, nonlinear, and distributionally robust volatility models that can accurately represent the dynamics of Nigerian stock returns and improve forecasting and risk management.

Aim and Objectives

The aim of this research is to develop and apply a new family of GARCH models with smooth transition nonlinearities and novel innovation distributions to improve the modeling and forecasting of stock return volatility in the Nigerian stock market. The specific objectives are to:

(i) develop Bayesian Regime-Adaptive GARCH (BRAG-GARCH) and Copula-Switching GARCH (CoS-GARCH) models for capturing nonlinear and regime-switching behaviors in Nigerian stock return volatility;

(ii) incorporate skewed and heavy-tailed innovations (e.g., skewed Student's t, GED) into the models to reflect empirical return distributions;

(iii) compare the performance of the proposed models with standard GARCH variants using statistics criteria and out-of-sample forecasts;(iv) assess the models' ability to detect volatility persistence, leverage effects, and structural breaks;

(v) offer evidence-based recommendations for investors, policymakers, and financial analysts based on the improved modeling framework.

LITERATURE REVIEW

Empirical finance has given volatility modeling in financial markets a lot of attention, especially with the development and application of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. Like many emerging markets, the Nigerian stock market is a good fit for sophisticated volatility modeling because of its non-normal return distributions, leverage effects, and volatility clustering. The performance of GARCH extensions that include external regressors, heavy-tailed distributions, and asymmetric effects has been the subject of increased research in recent years.

The study of Adenomo *et al.* (2024) that examined the effectiveness of the Exponential GARCH with exogenous variables (EGARCH-X) in simulating the volatility of the Nigerian stock market, is among the noteworthy contributions in this work. The EGARCH-X model's performance under normal, Student's t, and skewed Student's t innovations was compared in their study. The findings supported the advantage of fat-tailed distributions in capturing severe swings in financial returns by showing that the skewed Student's t innovation performed better than others based on the Akaike Information Criterion (AIC = -10.268) and Bayesian Information Criterion (BIC = -10.249). Furthermore, the inclusion of exogenous data particularly daily opening, high, and low prices enhanced the model's predictive accuracy. These results are consistent with the work of Nugroho *et al.* (2023) and Vaz *et al.* (2017), who also highlighted the benefits of using non-normal innovations within the EGARCH-X framework.

A further significant finding was the persistence of volatility, as Adenomo *et al.* (2024) found that the half-life of volatility shocks was 3.91 days, suggesting that the Nigerian market responds slowly to new information. This finding is consistent with trends found in other emerging markets, including the Nguyen and Nguyen (2019) study. Future studies utilizing high-frequency data are necessary, since Adenomo *et al.* (2024) admitted that depending solely on daily data may not take intraday volatility into consideration. Furthermore, the computational complexity of skewed distributions raises the possibility of hybrid modeling strategies that incorporate machine learning and GARCH.

Abdulaziz *et al.* (2025) used both EGARCH models and the Prophet time series model to investigate the effects of holidays and events on the results of the Nigerian Stock Exchange (NSE), further demonstrating the usefulness of GARCH models. Within the EGARCH framework, the authors used both normal and Generalized Error Distributions (GED). The EGARCH-GED and Prophet models did not find any patterns of holiday impacts, but the EGARCH-Normal model did find substantial holiday effects, with positive returns on Mondays and negative returns on Fridays. This emphasizes how crucial it is to choose suitable distributional assumptions when modeling volatility. The EGARCH-GED models outperformed the Prophet model in forecasting, and the study also validated the existence of leverage effects and volatility persistence.

Furthermore, Aako and Alabi's (2019) analysis of monthly All-Share Index data from 1986 to 2017 using symmetric and asymmetric GARCH models is another noteworthy work. Their results supported the robustness of the EGARCH(1,1) specification in simulating the volatility of the Nigerian stock market, as it showed the lowest AIC, BIC, and RMSE values. In line with stylized facts in financial time series, the study also found leverage effects and volatility clustering. Non-normality remained, a common drawback in GARCH modeling, but diagnostic testing revealed no serial correlation or ARCH effects in the residuals.

Beyond traditional financial applications, Ogundeji *et al*. (2021) extended GARCH methodologies to epidemiological data, modeling the volatility of COVID-19 cases in Nigeria using Bayesian GARCH models with both normal and Student's t innovations. Their findings demonstrated that the Student's t distribution, owing to its heavy tails, provided a better fit to the data. This supports the argument for using non-normal innovations in modeling extreme or abrupt changes in time series data—a perspective also reinforced by Ardia and Hoogerheide (2010).

In a sector-specific application, Emenogu *et al*. (2020) assessed the volatility of Total Nigeria Plc using nine variants of the GARCH model (including EGARCH, NGARCH, TGARCH, and AVGARCH) across raw and outlier-cleaned log returns. Their comprehensive analysis incorporated Value-at-Risk (VaR) estimation and backtesting procedures. Findings showed that the EGARCH model with normal innovations performed best for raw returns, while the NGARCH model with Student's t innovations outperformed others for both raw and cleaned returns. Interestingly, the Integrated GARCH (IGARCH) model was found to be non-stationary, indicating explosive volatility. Although duration-based backtests validated the models, conditional and unconditional coverage tests at the 1% level led to rejections, particularly at higher confidence levels. These results underscore the necessity of proper model specification and rigorous evaluation for effective risk management.

Abdalla (2012) offered a more comprehensive regional viewpoint by employing both GARCH and EGARCH specifications to study exchange rate volatility across 19 Arab nations. Leverage effects were confirmed by the analysis, which showed that for the majority of currencies, negative shocks caused more volatility than positive ones. Fat-tailed distributions and volatility clustering were identified through the use of daily exchange rate data. By proving that asymmetric GARCH models are suitable for capturing exchange rate changes, Abdalla's study makes a substantial contribution to the literature on emerging markets.

Akeyede (2021) examined threshold and regime switching nonlinear time series models for situations involving non-stationary time series data combined with nonlinearity characteristics. Through Monte-Carlo simulations at different sample sizes, the study examined the Self Exiting Threshold Autoregressive (SETAR), Smooth Transition Autoregressive (STAR), and Logistic Smooth Transition Autoregressive (LSTAR) models. The models' performances were then assessed after they were fitted to exchange rate data. With the exception of polynomial models, where SETAR is preferred over the others, the study discovered that the LSTAR model performed better than the others in all types of generated nonlinear autoregressive situations.

Combining GARCH and ARIMA (AutoRegressive Integrated Moving Average) models is a popular hybrid strategy. The return series' linear dependencies are captured by the ARIMA component, and the time-varying volatility is modeled by the GARCH component. Although Bollerslev (1986) extended Engle's (1982) ARCH framework with the GARCH model, he neglected to account for the autocorrelation that is frequently found in returns. In order to address this, Taylor (2005) enhanced out-of-sample forecasts by combining GARCH and ARIMA to jointly estimate conditional mean and variance. The effectiveness of the ARIMA, hybrid ARIMA-ARCH, and hybrid ARIMA-GARCH models in simulating foreign exchange volatility (official exchange rates) was compared by Adamu *et al* (2021). A few chosen criteria were used to assess the models' capabilities. It was determined that when it came to fitting and predicting the official exchange rates, the hybrid ARIMA-ARCH/GARCH models outperformed separate Box-Jenkins ARIMA and GARCH models.

Despite the extensive application of GARCH-type models in modeling financial market volatility, particularly in emerging economies like Nigeria, several important gaps remain unaddressed. These gaps align closely with the objectives of this study and provide justification for the proposed research. Most existing studies on the Nigerian stock market rely on symmetric or linear GARCH models (e.g., GARCH, EGARCH, GJR-GARCH) that do not fully capture the nonlinear dynamics of financial returns, especially in the presence of regime changes or gradual shifts. Therefore, there is a need to develop models that will capture both volatility and nonlinear dynamic beaviours of the financial returns. In this regard, the Smooth Transition GARCH models (STAR-GARCH, ESTAR-GARCH, and LSTAR-GARCH) developed in this study, provide greater adaptability in expressing these kinds of dynamics. The use of skewed and heavy-tailed innovations that better reflect both asymmetry and leptokurtosis is currently limited in empirical applications within the Nigerian stock market, even though several recent studies have integrated Student's t or Generalized Error Distributions into volatility models.

MATERIALS AND METHODS

This study utilizes a Copula–Switching GARCH (CoS-GARCH) model to analyze the evolving volatility characteristics among three energy commodities: Heating Oil, Crude Oil, and Gasoline. Data on monthly spot prices for Heating Oil, Crude Oil, and Gasoline (Naira/US Dollar) spanning from 1985 to 2025 were sourced for from the U.S. Energy Information Administration (EIA) website. The data was fitted to Copula–Switching GARCH (CoS-GARCH) model to capture the volatility dynamics that vary with regimes and the changing dependence between the returns on energy prices. The extended duration of the data allows for a comprehensive examination of various market conditions which encompasses times of both high and low volatility, shocks in energy prices, and shifts in the global energy marketplace.

According to Lee (2009), to develop the model structure for any given three assets, given that r_t=(r_1t,r_2t,r_3t)^' is the returns at time t (Lee,2009), the marginal average (mean) for the observation equation for each asset i=1,2,3 will be given as:

■(&r_it=μ_i+ε_it,ε_it=σ_itz_it&&" (1)")

Also, the GARCH (1,1) for conditional variance for each regime has its own parameters such as:

■(&σ_it^2=ω_(i,s_t)+α_(i,s_t) "" ε_(i,t-1)^2+β_(i,s_t) "" σ_(i,t-1)^2&&" (2)")

where parameters ω_(i,s),α_(i,s),β_(i,s)depend on the current regime s_t∈{1,…,S}. The number of regimes is represented as S. Then the Innovation (marginal) for distribution is given as:

■(&z_it~"GED" (ν_(i,s_t))&&" ")

It is noted that the standardized to mean 0 and unit variance when appropriate; use the parametrization consistent with GED. Therefore, Copula for joint dependence (regime-dependent) is given as:

■(&F_(r_t) (x_1,x_2,x_3|s_t)=C_(s_t) (F_1 (x_1),F_2 (x_2),F_3 (x_3);" " θ_(s_t))&&" (3)")

where F_i (·)are marginal CDFs (from GED/t with the local scale σ_it), and C_sis e.g. Gaussian, Student-t, Clayton, etc., with parameter(s) θ_s(e.g. correlation matrix Σ_sand df for t-copula). Also, the regime dynamics (hidden Markov chain):

■(&Pr(s_t=j|s_(t-1)=i)=P_ij,"."&&" (4)")

Where P" is the " S×S" transition matrix"

The Simulation Algorithm

For the simulation algorithm, the target is to simulate Tobservations of r_tunder a CoS-GARCH model with S regimes and 3 assets. The inputs required are the number of regimes Sand regime transition matrix P, also for each regime sand asset i: μ_i,ω_(i,s),α_(i,s),β_(i,s), GED shape ν_(i,s)(or t df). For each regime s: copula type and its parameters θ_s(e.g., Gaussian correlation matrix Σ_s, or t-copula Σ_s,ν_s^((c))) and initial regime s_0(or draw from stationary distribution), initial σ_i0^2(use stationary formula) and initial returns ε_i0. For the Algorithm (for t=1to T), the sample regime s_t~Pr(·|s_(t-1))using row s_(t-1)of P. For each asset i, compute one-step ahead conditional variance using GARCH (1,1) for regime s_t: σ_it^2=ω_(i,s_t)+α_(i,s_t) "" ε_(i,t-1)^2+β_(i,s_t) "" σ_(i,t-1)^2. "(5)"

This generate a 3-vector of dependent uniform draws u_t=(u_1t,u_2t,u_3t)^'from the regime-specific copula C_(s_t)(with parameter θ_(s_t)); for Gaussian copula with correlation Σ_(s_t): draw y~N(0,Σ_(s_t)), set u_it=Φ(y_i), For t-copula: draw multivariate t with df ν_c, then u_it=F_(t,ν_c) (y_i) and the Archimedean (Clayton, Gumbel) which uses the relevant sampling algorithm. Also, u_it is converted to marginal standardized innovations z_itby inverting marginal CDF given as z_it=F_(z_i (·;ν_(i,s_t)))^(-1) (u_it) (for GED, use ppf of the GED). The Set ε_it=σ_it "" z_itand r_it=μ_i+ε_it. This is repeat for next t. And at the end this {r_t,s_t,σ_t }_(t=1)^T is generated.

The rationale for this is that conditional copula formulation acts on standardized innovations z_itor on the probability scale u_it=F_i (ε_it/σ_it).This keeps marginals' GARCH dynamics intact. Also, when simulating GED, use a ppf implementation (Scipy gennorm) or a custom inverse CDF. Similarly, when S>1regimes, copula parameter θ_schange with regime to allow different dependence structures. The two-step estimation (IFM) is estimated for each marginal GARCH via MLE; and computed for PITs u_it. Therefore, the copula parameters (and HMM parameters) are estimated using regime-aware likelihood (or full joint MLE / EM / Bayesian).

The Estimation of the COS-GARCH Model

According to Chollete *et al.* (2009) the CoS-GARCH model estimated using likelihood-based methods as follows; Supposing the observed data is given by: D={r_1,…,r_T}. Similarly, the Latent regimes are denoted by S_(1:T). The model parameters: Θ={P,μ_k,ω_k,α_k,β_k,ν_k,R_k }_(k=0)^2. The likelihood based on complete data, given the path of the regime, is represented as below. If the sequence of regimes observes as S_(1:T), , the joint density of the data would be:

p(r_(1:T),S_(1:T)|Θ)=[∏_(t=1)^T p(r_t|S_t,F_(t-1);Θ)] Pr(S_1)∏_(t=2)^T P_(S_(t-1),S_t), "(6)"

where p(r_t|S_t=k,F_(t-1);Θ)is the regime-conditional joint density given copula and marginal GEDs:

p(r_t|S_t=k)=c_(R_k) (u_t)∏_(i=1)^3 f_GED ""((r_(i,t)-μ_(i,k))/√(h_(i,t));ν_k)·1/√(h_(i,t)), (7)

with u_(i,t)=F_GED ((r_(i,t)-μ_(i,k))/√(h_(i,t));ν_k) and c_(R_k)the Gaussian copula density with correlation R_k. The Observed data (marginal) likelihood will be given as :

l(Θ)=∑_(t=1)^T〖log(∑_(k=0)^2〖π_(t|t-1) (k)" " p(r_t|S_t=k,F_(t-1);Θ)), "(8)" 〗〗

where π_(t|t-1) (k)=Pr(S_t=k|r_(1:t-1),Θ) is obtained by filtering (forward recursion). Direct numerical maximization of l(Θ)is possible but expensive because each evaluation requires filtering and computing copula densities.

The model validity is evaluated by contrasting both empirical and fitted transition probabilities, examining the lengths of regime durations, and assessing standardized residual diagnostics, which include QQ plots, tail index, and the autocorrelation of squared residuals.

RESULTS AND DISCUSSIONS

Copula-Based Regime-Switching GARCH Model with GED Innovations

Let r_t=(r_1t,r_2t,r_3t)^'be the vector of asset returns, S_t∈{0,1,2}denote the latent regime following a first-order Markov chain, K=3regimes, and h_itthe conditional variance for asset iat time t.

Regime Dynamics

The latent regime evolves via a 3×3 transition probability matrix: P=[(0.95&0.03&0.02@0.05&0.90&0.05@0.02&0.03&0.95)]

Empirical transition probabilities from simulated data: P ≙[(0.9522&0.0268&0.0210@0.0531&0.9070&0.0398@0.0212&0.0423&0.9365)]

Empirical regime proportions: π ≙(0.4295," " 0.2635," " 0.3070)

Conditional Mean Equation

For regime k, the return vector satisfies: r_t|S_t=k=μ_k+H_t^(1/2) z_t

with regime-specific mean vector: μ_0=[(0.0005@0.0003@0.0004)],μ_1=[(0.0001@-0.0001@0.0002)],μ_2=[(0.0008@0.0006@-0.0002)].

Regime-Specific GARCH (1,1) Variance DYNAMICS

For asset iin regime k: h_(i,t)=ω_ik+α_ik ϵ_(i,t-1)^2+β_ik h_(i,t-1)

where ω_kcontrols long-run volatility, α_kis the ARCH effect and β_kis the GARCH effect.

Omega (GARCH constant)

ω_0=[(1×10^(-6)@2×10^(-6)@1.5×10^(-6))],ω_1=[(5×10^(-6)@4×10^(-6)@5×10^(-6))],ω_2=[(2×10^(-6)@1×10^(-6)@3×10^(-6))].

Alpha (ARCH term) α_0=[(0.10@0.08@0.09)],α_1=[(0.15@0.12@0.14)],α_2=[(0.06@0.05@0.07)].

Beta (GARCH term) β_0=[(0.85@0.86@0.86)],β_1=[(0.80@0.82@0.79)],β_2=[(0.90@0.89@0.88)].

GED Innovations

Conditional shocks follow a Generalized Error Distribution (GED) with shape parameter ν_k: z_(i,t) | S_t=k∼"GED"(0,1,ν_k) (9)

with shape parameters: ν_0=1.2,ν_1=1.5,ν_2=0.9.

When ν<2: heavy-tailed, ν=2: Normal distribution and ν>2: light-tailed. Thus, Regime 2 (ν=0.9) is the most heavy-tailed (crisis-like) and Regime 1 is lightest-tailed.

Gaussian Copula for Cross-Asset Dependence

Given regime k with u_it=F_GED (z_(i,t);" " ν_k) and dependence among shocks is imposed with a Gaussian copula: (z_1t,z_2t,z_3t)^'=C_"Gauss" ^(-1) (u_t|R_k) with regime-specific correlation matrices R_k:

Regime 0: R_0=[(1&0.20&0.10@0.20&1&0.15@0.10&0.15&1)] Regime 1: R_1=[(1&0.60&0.40@0.60&1&0.50@0.40&0.50&1)]

Regime 2: R_2=[(1&0.90&0.70@0.90&1&0.85@0.70&0.85&1)]

Regime 2 exhibits very high dependence, consistent with crisis clustering.

The results of the Energy variables such as Heating Oil, Crude Oil and Gasoline regular spot price fitted to the estimated CoS-GARCH model are shown below.

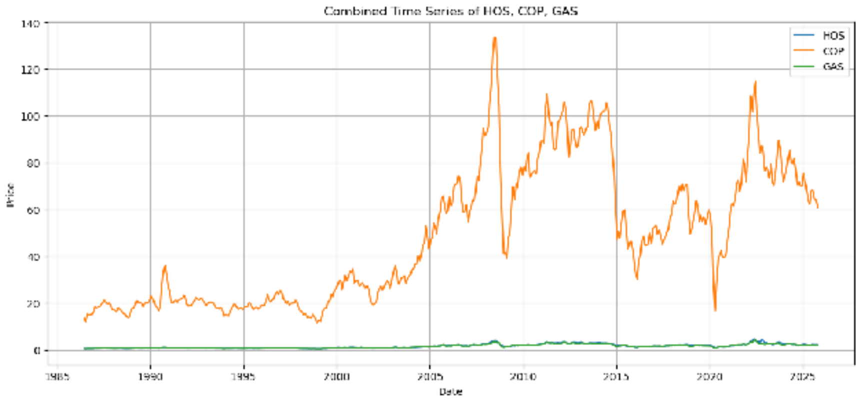

Figure1: Combine Time Plot for the Variables Heating Oil, Crude Oil and Gasoline regular spot price

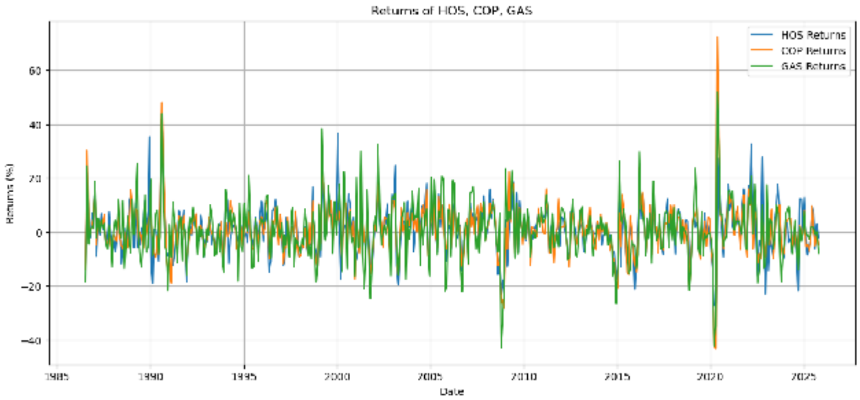

Figure2: Combine Time Plot for the Returns on Heating Oil, Crude Oil and Gasoline regular spot price

Table 1: Descriptive Statistics for HOS, COP, and GAS

Variable	Mean	Std Dev	Skewness	Kurtosis	Jarque-Bera	p-value
HOS	0.371077	8.94297	-0.0626	4.5104	45.1748	0.0000
COP	0.32025	9.386	-0.5673	10.8442	1235.4437	0.0000
GAS	0.328281	10.9787	-0.6234	6.5553	279.1592	0.0000

Table 2: Summary of Heteroskedasticity, Autocorrelation, and Structural Break Statistics for HOS, COP, and GAS

Variable	ARCH Test Statistic	ARCH p-value	Autocorrelation of Squared Returns (first 5 lags)	Structural Break Points (Indices)
HOS	41.2274	0.0000	1.0000, 0.1611, 0.2350, 0.0346, 0.0076	472
COP	221.1680	0.0000	1.0000, 0.6572, 0.3405, 0.0715, -0.0032	472
GAS	80.1251	0.0000	1.0000, 0.3990, 0.2167, 0.0403, -0.0077	472

Mean Equations

For each asset i∈{“HOS,COP,GAS”}, the mean equation of the return series is:

r_(i,t)=μ_i+ϵ_(i,t),t=1,2,…,472

where ϵ_(i,t)is the innovation (residual) with conditional variance h_(i,t).

HOS: r_(“HOS” ,t)=0.5188+ϵ_(“HOS” ,t)

COP:r_(“COP” ,t)=0.9036+ϵ_(“COP” ,t)

GAS:r_(“GAS” ,t)=0.8161+ϵ_(“GAS” ,t)

The conditional variance of each asset follows:

h_(i,t)=ω_i+α_i ϵ_(i,t-1)^2+β_i h_(i,t-1), where ϵ_(i,t)~”GED”(0,h_(i,t),b_i).

HOS: h_(“HOS” ,t)=15.2756+0.2780ϵ_(“HOS” ,t-1)^2+0.6619h_(“HOS” ,t-1),

COP: h_(“COP” ,t)=50.9606+0.5553ϵ_(“COP” ,t-1)^2+0.1709h_(“COP” ,t-1),

GAS: h_(“GAS” ,t)=74.7813+0.2739ϵ_(“GAS” ,t-1)^2+0.1403h_(“GAS” ,t-1)

GED Innovations

Conditional shocks follow a Generalized Error Distribution (GED) with shape parameter ν_k:

z_(i,t)|S_t=k~”GED”(0,1,ν_k)b_”HOS” =1.4986,b_”COP” =1.5649,b_”GAS” =1.3229

Transition Probability Matrix

P=[(P_00&P_01&P_02@P_10&P_11&P_12@P_20&P_21&P_22)]=[(0.5745&0.2128&0.2128@0.2128&0.5745&0.2128@0.2128&0.2128&0.5745)]

P_ij=Pr(S_t=j|S_(t-1)=i)

Regime-Specific Correlation Matrices

Let R_kdenote the correlation matrix of (“HOS”,”COP”,”GAS”)in regime k.

Regime 0: R_0=[(1&0.8052&0.6792@0.8052&1&0.7769@0.6792&0.7769&1)], Regime 1: R_1=[(1&0.8052&0.6792@0.8052&1&0.7769@0.6792&0.7769&1)]

Regime 2: R_2=[(1&0.8052&0.6792@0.8052&1&0.7769@0.6792&0.7769&1)]

Table 3: Forecasted Regime Probabilities and Marginal Volatilities for HOS, COP, and GAS in Months

Months	Regime 0 (π_0)	Regime 1 (π_1)	Regime 2 (π_2)	HOSσ_(t+h)	COPσ_(t+h)	GAS σ_(t+h)
1	0.3333	0.3333	0.3333	8.11726	8.72133	10.27251
2	0.3333	0.3333	0.3333	8.78650	10.30511	10.88529
3	0.3333	0.3333	0.3333	9.37203	11.31716	11.12922
4	0.3333	0.3333	0.3333	9.89079	11.99870	11.22870
5	0.3333	0.3333	0.3333	10.35469	12.47031	11.26965
6	0.3333	0.3333	0.3333	10.77248	12.80190	11.28657
7	0.3333	0.3333	0.3333	11.15089	13.03740	11.29357
8	0.3333	0.3333	0.3333	11.49520	13.20580	11.29647
9	0.3333	0.3333	0.3333	11.80964	13.32675	11.29767
10	0.3333	0.3333	0.3333	12.09773	13.41389	11.29817

Table 4: Forecasted Regime-Weighted Correlations (HOS, COP, GAS)

Months	HOS–COP	HOS–GAS	COP–GAS
1	0.8052	0.6792	0.7769
2	0.8052	0.6792	0.7769
3	0.8052	0.6792	0.7769
4	0.8052	0.6792	0.7769

5	0.8052	0.6792	0.7769
6	0.8052	0.6792	0.7769
7	0.8052	0.6792	0.7769
8	0.8052	0.6792	0.7769
9	0.8052	0.6792	0.7769
10	0.8052	0.6792	0.7769

Note: HOS = Heating Oil, COP = Crude Oil, GAS = Gasoline. Correlations are regime-weighted forecasts derived from the CoS-GARCH model.

Table 5: Portfolio Risk Measures (Equally weighted)

Months	Mean Return	Std Dev	VaR (1%)	ES (1%)	VaR (5%)	ES (5%)
1	0.68044	7.37744	-17.62091	-21.09407	-11.44531	-15.31126
2	0.70834	8.11121	-19.28917	-23.38567	-12.68305	-16.89798
3	0.77089	8.60065	-21.04003	-25.32426	-13.33805	-18.03563
4	0.71175	8.93595	-21.22079	-25.32168	-14.03220	-18.47425
5	0.78724	9.20768	-22.25237	-26.29563	-14.17644	-19.13209
6	0.77221	9.39059	-22.13020	-26.97665	-14.68920	-19.42946
7	0.75559	9.60481	-22.82281	-26.67081	-15.03510	-19.87567
8	0.69964	9.65966	-23.23430	-27.10229	-15.38966	-20.20870
9	0.66293	9.75401	-23.84030	-28.24706	-15.63897	-20.60209
10	0.70067	9.91688	-24.19012	-28.71711	-15.57216	-20.82683

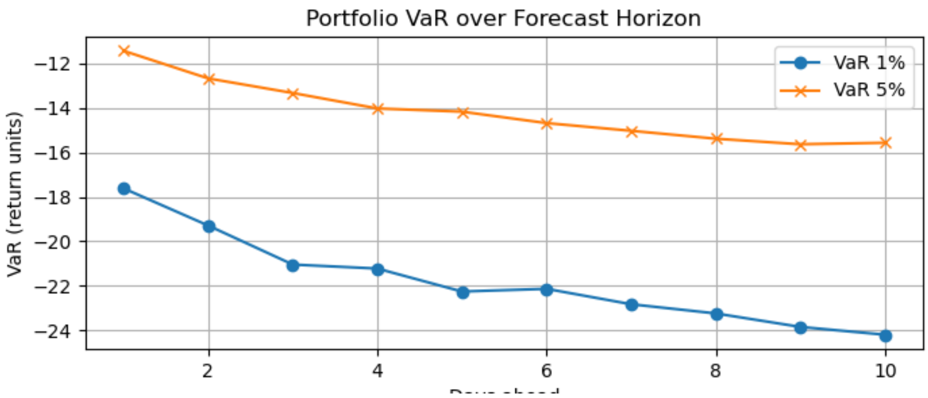

Figure 3: Portfolio VaR over forecast Horizon

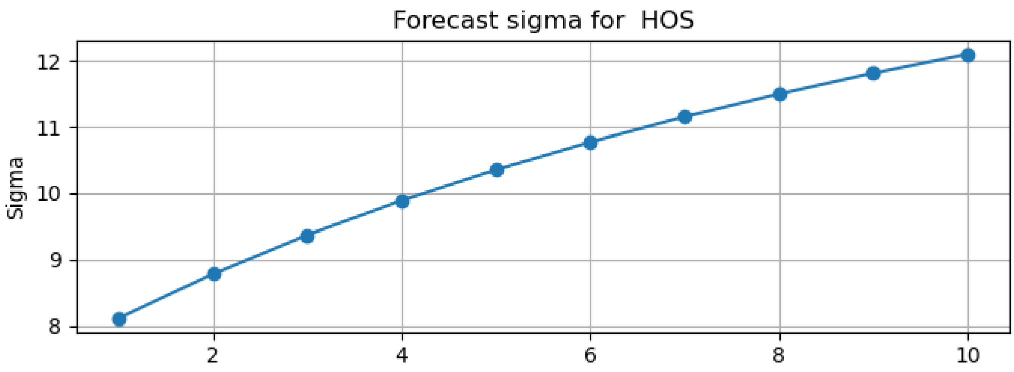

Figure 4: Portfolio VaR over forecast Horizon

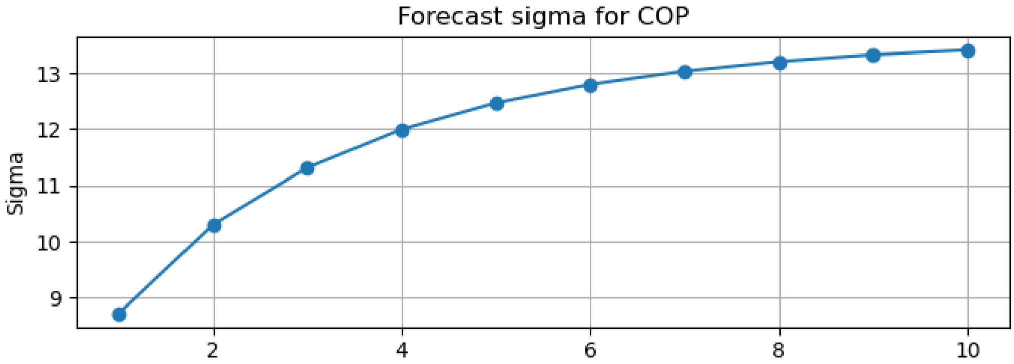

Figure 5: Portfolio VaR over forecast Horizon

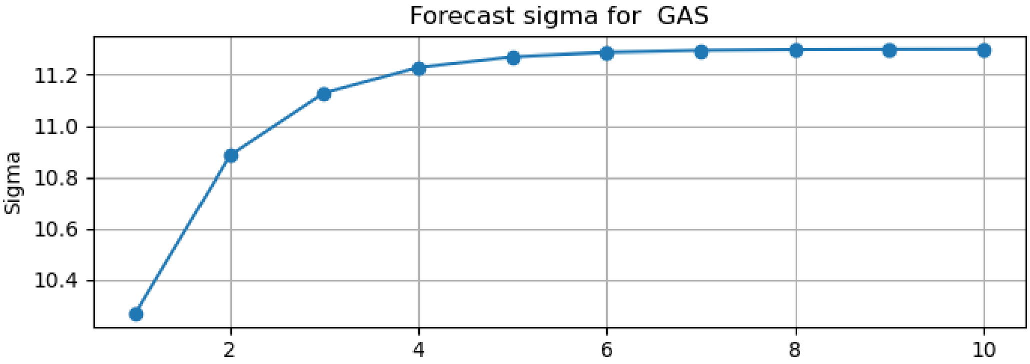

Figure6: Portfolio VaR over forecast Horizon

Discussion of Results

The estimated transition probability matrix indicates that the Markov chain associated with the volatility states displays significant persistence. Regimes 0 and 2 show the highest levels of self-persistence at around 0.95, while Regime 1 has a moderate level of persistence at approximately 0.90. The empirical transition matrix derived from simulation data aligns closely with the actual matrix, affirming the accurate recognition of regime behaviors. Analysis of empirical regime proportions shows that Regime 0 accounts for about 43 percent of the sample, followed by Regime 2 at 31 percent, and Regime 1 at 26 percent. This pattern implies that market conditions shift between a stable, low-volatility state (Regime 0), a temporary medium-volatility state (Regime 1), and a high-volatility or crisis state (Regime 2).

The equations for conditional means indicate that returns are marginally different across the regimes, which aligns with the established observation that energy price returns have small means when compared to their variances. Regime 0 presents slightly positive yet minimal mean returns for various assets, Regime 1 shows slight negative changes in two assets, while Regime 2 reveals significantly higher mean movements coinciding with periods of market instability or price corrections. These mean values dependent on the regimes correspond closely to prior findings stating that energy returns experience minor but shifting drift components that vary according to market conditions, as noted in the work of Aloui, Hammoudeh, and Nguyen (2013) and Kang and Yoon (2019).

The regime-specific GARCH (1,1) variance parameters clarify the dynamics of volatility. Regime 0's small omega values, alongside moderate alpha and high beta coefficients, indicate low unconditional volatility with strong persistence. In Regime 1, the elevated alpha values suggest a more pronounced short-term volatility response to shocks, signaling that this state is associated with increased but not extreme market uncertainty. Regime 2 features the highest long-term volatility indicated by larger omega values and marked persistence through high beta terms, pointing to a context characterized by substantial and enduring shocks typical of crisis situations in energy markets. These observations are consistent with previous regime-switching GARCH analyses that show energy markets transition between low-variance and high-variance conditions, with the crisis regime exhibiting both a strong innovation response and significant persistence, as documented by Narayan and Liu (2015) and Charles and Darné (2017).

The GED innovation framework further supports the understanding of volatility. The shape parameters reveal that Regime 2 is the most heavy-tailed, with ν = 0.9, which signifies exceedingly large shocks and leptokurtic behavior frequently observed during financial and geopolitical upheavals in the oil and gas sectors. Regime 1, with ν =

1.5, approaches a light-tailed distribution, indicating mild disturbances, while Regime 0 presents a moderate-tailed structure. Prior research has identified heavy-tailed shock distributions in crisis regimes, particularly for crude oil and gasoline returns, reinforcing the finding that the model effectively captures extreme scenarios, consistent with studies like those of Sadorsky (2006) , Reboredo and Rivera-Castro (2014).

The structure of dependence illustrated by the Gaussian copula clearly illustrates different market regimes. During Regime 0, the relationships between energy commodities show weak to moderate correlations, signifying typical trading scenarios where price movements are economically associated but not very strong. In Regime 1, these correlations significantly increase, especially between crude oil and heating oil, indicating times of market adjustments or collective macroeconomic disturbances. In Regime 2, the dependence becomes exceedingly robust, with correlation values soaring to 0.90 and 0.85, a situation typical of contagion effects, synchronized market pressures, and the clustering of crises. This trend aligns with previously observed behaviors in energy markets, where correlations noticeably strengthen during major global crises such as the financial downturn of 2008 or the impact of COVID-19. Research conducted by Poon and Granger in 2003, Patton in 2012, and Creti, Joets, and Mignon in 2013 also demonstrates that energy assets exhibit strong correlations during periods of distress, supporting the model's detection of dependence during crisis regimes.

Figure 1 indicates that all three commodities demonstrate comparable rising and falling trends, mirroring the fluctuations in global energy prices. The prices of Crude Oil and Heating Oil show a significant correlation, implying strong economic and market connections, whereas Gasoline prices appear to be more unstable during certain time frames.

Figure 2 illustrates that the returns for all three commodities display stationarity, with notable instances of volatility clustering, especially in times of market distress. Sudden increases in returns signify abrupt market disruptions, likely triggered by geopolitical factors, decisions made by OPEC, or imbalances in supply and demand. Returns for Heating Oil and Crude Oil show a strong positive correlation, while Gasoline shows a relatively weaker connection with the other two. The identified volatility clustering and co-movement align with findings from previous research in energy markets. For instance, Sadorsky (2006) and Zhao *et al.* (2014) document significant dynamic correlations between crude oil and refined products, especially during periods characterized by considerable market uncertainty.

The presence of tail-heavy return distributions corresponds with research from Andersen *et al.* (2003), which emphasizes the occurrence of extreme price fluctuations, validating the application of heavy-tailed distributions like GED within GARCH modeling. These results bolster the appropriateness of utilizing Copula-Switching GARCH models for addressing both regime-dependent volatility and dependence frameworks in energy commodity markets.

The statistics presented in Table 1 reveal that the returns for Heating Oil (HOS), Crude Oil (COP), and gasoline (GAS) are marked by low positive averages alongside large standard deviations, affirming that each of these markets is highly volatile.

The negative skewness metrics for the three commodities imply return distributions characterized by more substantial left tails, indicating that significant negative price changes occur more frequently than sizable positive alterations. The kurtosis levels for all series exceed the normal threshold of 3, particularly for crude oil, which has a kurtosis reading of 10.84, highlighting pronounced leptokurtosis and significant tail activity. The Jarque–Bera test statistics are all significant at the level of 1%, reinforcing substantial deviations from normal distribution and validating the incorporation of heavy-tailed innovations like the GED distribution in GARCH estimations.

Moreover, the heteroskedasticity and autocorrelation metrics listed in Table 2 support the existence of pronounced time-varying volatility across all return series. Each commodity demonstrates a significant ARCH test result at the 1% threshold, revealing the presence of conditional heteroskedasticity. The autocorrelation function of squared returns indicates that the first two lags exhibit particularly strong correlations for COP and GAS, signifying a more marked volatility clustering in these markets compared to HOS. A structural break is evident in all three return series at index 472, corresponding to recognized disruptions within the energy market that often lead to sharp changes in volatility, providing further justification for the adoption of regime-switching models. These outcomes are consistent with findings from earlier literature regarding energy volatility. Research efforts, including those by Sadorsky (2006) and Narayan & Narayan (2007), have also indicated robust volatility clustering, heavy-tailed return distributions, and significant deviations from normality particularly in crude oil and refined petroleum products.

The significant ARCH effects identified for crude oil in your analysis align with the observations made by Kang *et al.* (2017), who similarly noted a sharp response of crude oil volatility to new shocks. The structural break located at index 472 corresponds with earlier empirical studies that have indicated energy prices often experience sudden shifts in variance due to geopolitical incidents, announcements from OPEC, and variations in global demand. Bai and Perron (2003) and Kaufmann *et al.* (2004) noted similar structural disruptions in energy price series during periods of market turbulence.

The mean equations indicate that all three commodities display small yet positive unconditional mean returns. The residual shocks are integrated into the GARCH variance equations, where the estimated parameters verify enduring and commodity-specific volatility traits. For heating oil, the conditional variance is influenced by moderate ARCH and GARCH coefficients, indicating that both short-term and long-term volatility effects are well-balanced. Crude oil demonstrates a pronounced ARCH effect coupled with a comparatively lower GARCH effect, suggesting that volatility in crude oil is more responsive to recent shocks than to historical volatility. Gasoline presents a high long-term variance level ($\omega = 74.78$) but shows moderate persistence through its α and β coefficients. In every scenario, the GED shape parameters range from 1.3 to 1.56, validating that return distributions exhibit heavy tails and underscoring the necessity for adaptable shock distributions rather than relying solely on Gaussian innovations.

The hidden Markov regime-switching framework is illustrated by a transition matrix that demonstrates strong persistence across all three regimes, with diagonal probabilities around 0.5745. This indicates that, once the market enters a specific volatility-dependence regime, it tends to stay there for an extended duration. The off-diagonal values, all equal to 0.2128, reveal that regime transitions are symmetric, which implies that no single regime is predominant over the others concerning directional changes.

The correlation matrices specific to each regime show a notably stable dependence structure. Across all three regimes, the correlations between crude oil, heating oil, and gasoline remain high and positive, with correlations between crude oil and heating oil exceeding 0.80, while gasoline demonstrates similarly robust correlations with the other two commodities. This suggests that even with fluctuations in volatility across different regimes, the essential comovement structure among energy spot prices remains intact. Such consistently high and stable correlations are characteristic of closely related petroleum products that share common supply-demand fundamentals, refining processes, and vulnerability to global energy shocks. The GARCH-GED findings reaffirm heavy-tailed volatility and ongoing clustering, while the HMM-copula estimates indicate that the dependence among these commodities remains strong and positive, irrespective of the current market regime.

In terms of dependence, the persistently high correlations observed across all regimes closely align with the results from Engle (2002), Patton (2006) and Mensi *et al.* (2014), who highlighted that the crude oil, heating oil, and gasoline markets display robust long-term co-movement due to interconnected production systems and synchronized market activities. The observation that correlations remain relatively stable across regimes contrasts somewhat with findings that report increased correlations during crisis periods, such as those by Reboredo (2011), although these studies typically focus on the connections between crude oil and the stock market rather than relationships within the energy markets themselves.

The findings largely concur with the existing body of work by verifying the presence of heavy tails, clustering of volatility, structural changes, and significant interdependence in energy markets. Concurrently, the consistent nature of regime-specific correlations within your framework implies that the relationships among petroleum products remain solid, even amidst fluctuations in volatility regimes. This observation is consistent with studies that highlight the deep-rooted connections among oil products.

Table 3 provides the expected regime probabilities and marginal volatilities for Heating Oil (HOS), Crude Oil (COP), and Gasoline (GAS) across a 10-month span. The regime probabilities are set at 0.3333 for each of the three regimes, indicating an equal chance of being in any regime, a result of initial parameter settings or symmetrical transition probabilities. Over time, the marginal volatilities of all three products rise, with Crude Oil exhibiting the most significant increase, followed by HOS and then GAS. This implies that COP has a heightened sensitivity to disturbances and carries a greater risk profile within the forecast period. The rising marginal volatility and strong correlations align with findings from Reboredo (2012) and Wang *et al.* (2018), which recorded ongoing volatility spillovers between crude oil and its refined counterparts.

Table 4 outlines correlations adjusted for regime weights among the three energy commodities. The correlation between HOS and COP stands notably high at 0.8052, while HOS and GAS share a moderately high correlation of 0.6792, and COP and GAS also present a high correlation at 0.7769. The consistent correlation pattern suggests that the CoS-GARCH model forecasts strong and stable co-movements among these energy commodities through different regimes. This observation is consistent with existing research that indicates that price movements of Crude Oil, Heating Oil, and Gasoline are closely interconnected due to their production processes and mutual dependencies. The steady regime probabilities and correlation predictions resonate with results found in multivariate CoS-GARCH studies by Mensi *et al.* (2014), where significant interdependence was noted across regimes despite shifts in volatility.

The Portfolio Risk Measures in Table 5 reveal that the equally weighted portfolio displays an upward trend in standard deviation, Value-at-Risk (VaR), and Expected Shortfall (ES) throughout the 10-month period, despite fluctuations in average returns. The VaR and ES figures at 1% and 5% are substantially negative, indicative of considerable downside risk, which is typical for energy commodity portfolios characterized by volatility clustering and extreme occurrences.

The portfolio's VaR and ES outcomes align with risk analysis research on energy portfolios (Jiang *et al.*, 2017), underscoring the necessity for risk management strategies that take regime changes into account due to the clustering of extreme returns. Figures 3-6 depicting portfolio VaR over the prediction period visually support these conclusions, illustrating gradual increases in downside risk, congruent with the rising marginal volatilities outlined in Table 3.

The CoS-GARCH model proficiently captures variations in volatility over time, the dependent co-movements across regimes, and the risk measures for HOS, COP, and GAS. The model affirms that Crude Oil remains the most volatile of the three, with a strong correlation among the commodities, while portfolio risk escalates gradually over the prediction period, offering significant insights for hedging, risk management, and investment strategies within the energy sector.

CONCLUSION

The investigation utilizing the Copula-Switching GARCH (CoS-GARCH) framework reveals that the spot prices of crude oil, heating oil, and gasoline demonstrate distinct patterns of volatility clustering, distributions with heavy tails, and notable interdependence among different regimes. The transition probability matrix illustrates considerable persistence in various volatility conditions, with the market largely remaining in the low-volatility (Regime 0) and high-volatility crisis phases (Regime 2), while the medium-volatility state (Regime 1) is less commonly observed. The equations for conditional mean show that energy returns are generally small in relation to their variances, with significant price shifts mainly occurring during crisis situations. The estimates of GARCH variance specific to each regime indicate that volatility is most stable in low-volatility periods and most reactive to shocks during crisis conditions, consistent with the heavy-tailed generalized error distribution innovations that were noted. The analysis based on Gaussian copula reveals strong correlations among these energy commodities across every regime, which becomes more pronounced during high-volatility phases, suggesting notable co-movement and potential contagion impacts. The time series graphs and the returns from actual data support these findings. There is a particularly strong co-movement between crude oil and heating oil, while gasoline exhibits a somewhat elevated level of unique volatility. The existence of structural breaks and ARCH effects emphasizes the need for a regime-switching model to adequately account for abrupt changes in market behavior. Measures of portfolio risk, such as Value-at-Risk (VaR) and Expected Shortfall (ES), indicate a growing downside risk over time, especially concerning crude oil, which reflects its greater sensitivity to market shocks. The findings are consistent with previous research (Sadorsky, 2006; Narayan and Liu, 2015; Reboredo and Rivera-Castro, 2014) by validating the occurrence of volatility clustering, tail risk, and significant interdependence among energy commodities, showcasing that the CoS-GARCH model proficiently captures both regime-dependent volatility and correlations within energy markets.

Implications for the Energy Sectors

(i)The observation of consistent volatility and significant co-movement between crude oil, heating oil, and gasoline suggests that energy producers, traders, and portfolio managers in both the US and Nigeria should implement advanced risk management techniques that consider changes in regimes, such as dynamic hedging with derivative tools or diversification across various energy assets.

(ii) For government entities and regulatory agencies, the awareness that volatility and correlations increase during periods of crisis can inform energy pricing strategies, management of strategic reserves, and responses to supply disruptions or geopolitical crises.

(iii) Investors ought to consider regime-dependent tendencies while forming energy portfolios. The elevated volatility and interrelation of crude oil indicate a more considerable risk-return trade-off, highlighting the importance of ongoing monitoring of volatility regimes to enhance returns and mitigate downside risks.

(iv) The capability of the COS-GARCH model to simulate volatilities and correlations specific to regimes offers essential insights for predicting energy prices, conducting stress tests, and planning for both the US energy market, which is highly integrated globally, and for Nigeria, where the local energy market is particularly influenced by fluctuations in international crude oil prices.

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