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Forecasting Nigeria's Oil Price Volatility: A Comparative Analysis of GARCH Models and Heston's Stochastic Models

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ABSTRACT

Modeling the volatility of crude oil prices is essential because it gives substantial influence to the oil producing countries. Nigeria, the biggest oil producer in Africa and a major participant in the world oil market, has significant economic difficulties changes in oil prices. This study uses 14 years of crude oil price data (2010–2023) to assess and compare the forecasting effectiveness of the Heston stochastic volatility model and GARCH-type models (GARCH, EGARCH, IGARCH, TGARCH, and FIGARCH). According to the analysis, GARCH-type models with Student's t-distribution perform better than models with typical innovation. With a log-likelihood value of 12022.3, an AIC of -4.7012, a mean error (ME) of 0.0254, and a root mean square error (RMSE) of 0.0534, the EGARCH model outperformed the others. Nonetheless, the Heston model outperformed all GARCH-type models in terms of forecast accuracy, achieving the smallest error (0.000564) and successfully capturing fat-tail characteristics in daily return distributions. The study indicates that the Heston model offers a better fit and more accurate forecast than GARCH-type models using data from January to December 2023 for out-of-sample forecasting. These results provide stakeholders and policymakers with important information for controlling the volatility of Nigeria's crude oil market.

INTRODUCTION

One of the most important energy sources in the world is petroleum, a fossil fuel that was created over millions of years from the remains of marine plants and animals. It is known as "crude oil" in its natural state. Crucially, crude oil serves as a raw material and a necessary energy source (Sekati *et al.*, 2020; Yi *et al.*, 2021; Dunn & Holloway, 2012). Common forms of oil that power cars, ships, and airplanes include heating oil, diesel, motor gasoline, and jet fuel. Oil is used in the production and transportation of many commonplace goods, and the industry that produces it has a big impact on other industries. Production expenses and the state of the economy as a whole are significantly impacted by changes in the price of petroleum products (Fondo *et al.*, 2021). Changes in oil prices impact many aspects of society, including household appliances, detergents, prescription medications, and food supplies. Like any commodity, oil prices fluctuate in response to supply and demand, which can have a favorable or negative effect on a number of economic sectors (Gasper & Mbwambo, 2023). Crude oil is therefore still a major economic issue and a hot topic in discussions about international economic policy. It is clear from the last three decades that the housing bubble-related financial crisis had a negative effect on oil prices, which fell from US$133.88 per barrel in June 2008 to less than $40 per barrel in the months after the disaster. Following that, prices rose to $100 in 2014 before sharply falling to $30 in 2016 as a result of an increase in the supply of crude oil. The start of the Covid-19 pandemic made matters worse and caused prices to drop to $16.55, the lowest level in 20 years, in April 2020. However, prices saw another increase as a result of the events in Ukraine. Countries that rely significantly on crude oil are surely impacted by these global changes in crude oil prices (Rodhan, 2023). The extraction and sale of crude oil is Nigeria's main source of revenue. Following years of exploration that started in 1938, Shell D'Arcy, now known as Shell Petroleum Company, made the first commercial oil discovery in 1956 near Oloibiri in Bayelsa State, according to the Nigerian National Petroleum Corporation (NNPC, 2013). With its first oil field going online in 1958 and producing 5,100 barrels per day, Nigeria's abundance of oil won it a prominent place in the world market. Foreign businesses were then allowed to explore for oil in Nigeria, which resulted in the oil industry's steady expansion and made Nigeria a world leader. Nigeria currently produces the most oil in Africa, accounting for 33–35 percent of the continent's oil and gas reserves, or 1.347 million barrels a day. It is the fifth-largest oil exporter to the United States of America and the fifth-largest exporter in the Organization of Petroleum Exporting Countries (OPEC) (Mary, 2023). Later, in 1977, the Nigerian National Petroleum Corporation (NNPC) was established with the intention of overseeing and taking part in the nation's oil industry (Nwokeji, 2007). The nation had 36.966 billion barrels of oil and condensate reserves in 2023. This amounts to 5.906 billion barrels of condensate and 31.060 billion barrels of oil. Nigeria produces more than 1.5 million barrels of oil per day, with a total oil reserve of 37.064 billion barrels (Nuprc, 2023). Nigeria ranked 11th on the

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list of nations with oil reserves that exceed one billion barrels (38.6%), with about 37 million barrels of known oil reserves. Nigeria produced over 1.93 million barrels of oil per day and exported 85% of its oil production, placing it 15th in the globe on the list of nations that produce oil that year (John, 2023). In order to diversify the economy, increase the domestic market, and lessen an excessive reliance on crude oil exports, Nigeria has taken a number of actions to reform the energy sector, including the Petroleum Industry Act in 2021 and the elimination of subsidies in 2023. The Nigerian government raised the price of petrol at the pump from the regulated price of N185 to more than N700 per liter in May 2023 and announced the elimination of the fuel subsidy. An unusual protest against the government's decision to raise the price of fuel pumps was sparked by this rise. The elimination of the Premium Motor Spirit (PMS) gasoline subsidy is one of the most controversial topics in Nigeria right now (Odewale, 2023). The subsidy is a type of price manipulation in which the government sets the pump price for consumers to purchase and reimburses the store for the difference between the official or regulated price per liter and the actual market price. Subsidies, in my opinion, are the additional funds that the government spent or incurred in order to lower the price of PMS pumps for those with lower incomes. Nigeria's level of living will be significantly impacted by the elimination of the petroleum subsidy, particularly for the already underprivileged populations. The price of gasoline pumps has increased as a result of the elimination of the petroleum subsidies, raising the expense of food, transportation, and other necessities. When compared year-over-year, the food inflation rate in May 2023 was 24.82%, 5.33% higher than the rate in May 2022 (19.50%). The transportation and storage industry, which contributes roughly 0.89% of the GDP, saw the biggest fall of any sector in the second quarter of 2023, contracting by an astounding 50.64%. The average fare passengers paid for bus trips within the city each drop rose by 97.88% from N649.59 in May 2023 to N1,285.41 in June 2023, according to data from the NBS Transportation Watch. It increased by 120.63% year over year from N583 in June 2022 (NBS, 2023). For Nigerians, floating the country's currency has socioeconomic ramifications as well, hurting residents' purchasing power and their capacity to pay for necessities. Strong social safety nets, focused interventions, and transition-protective legislation would be necessary to mitigate these consequences. Low-income families and individuals' budgets are strained as a result, which lowers their level of living. Additionally, rising gasoline prices have caused inflationary pressures throughout the economy, raising the cost of products and services. People with fixed incomes are disproportionately affected by inflation, which makes poverty worse. Low-income people bear a disproportionate amount of the burden of rising living expenses and petroleum pump prices, which exacerbates the nation's income inequality and widens the income gap (Yakubu *et al.*, 2023). It is commonly acknowledged that the fluctuations in oil prices have a substantial impact on economic activity. Commodity market price fluctuations are frequently influenced by changes in oil prices, which can cause economic slowdowns and price swings for other commodities when oil prices abruptly rise or fall. Accordingly, predicting the price of crude oil is an important field of study, although it faces inherent challenges including excessive volatility (Wang *et al.*, 2004). While limited liquidity and occasional trade in imperfect markets may cause a delay in responding to new information, oil prices may not always react immediately to it (Monoyios & Sarno 2002). According to this viewpoint, a substantial body of research has been done on enhancing econometric models' capacity to simulate oil prices. A portion of the research uses different GARCH models to examine the trajectory of oil prices. Alessandri and Mumtaz (2019), Popp and Zhang (2016), Adams *et al.* (2024), and Van Robays (2016) are only a few of the research that have demonstrated the substantial influence of economic factors on rising volatility, particularly during times of regime transition. The forecasting skills of single-regime models are significantly diminished by regime shifts, which are influenced by a variety of economic factors. Additionally, by upsetting the trends shown in economic time series, economic considerations have a major impact on business cycles. For example, the performance of econometric models was significantly impacted by the oil crises of 1974 and 1979. Because of this, conventional volatility models that do not consider regime-switching features, including those brought on by oil shocks, are no longer sufficient to simulate volatility in gasoline prices. GARCH-type models (Ahmed & Shabri, 2014; Wacuka Ng'ang'a & Oleche, 2022; Adams & Bello, 2022; Deebom & Essi, 2017), Support Vector Machines (SVM), which forecast data with high volatility (Okasha, 2014), and Autoregressive Integrated Moving Average (ARIMA), also known as the Box-Jenkins Methodology (Shambulingappa *et al.*, 2020; Awujola *et al.*, 2015; Rodhan & Jaaz, 2022) are some of the analytical techniques that have gained a lot of attention recently in crude oil forecasting. Since the symmetric models of the GARCH family are better at predicting the price of crude oil, they are regarded as fitted models (Haque *et al.*, 2021; Arachchi, 2018; Herrera *et al.*, 2018). They are seen as crucial for figuring out how volatile various commodities are (Charles & Darné, 2021). By taking into consideration the effects of leverage, volatility clustering, and leptokurtosis in the time series analysis, the GJR-GARCH model further sets itself apart from other forecasting models. Furthermore, it is discovered that both symmetric and asymmetric models are successful in capturing volatility (Ekong & Onye, 2017). A crucial part of many financial decision-making procedures is the examination of financial time series volatility. Building less risky portfolios, maximizing asset allocation, and increasing returns all depend on accurate volatility forecasts. As a result, accurate volatility forecasting and analysis have become more crucial in recent years. The best choice of volatility models, however, is hotly debated, which presents a problem for researchers because the choice has an immediate effect on their findings. In order to solve the problem of volatility forecasting, this paper suggests using stochastic models and GARCH-type models. This study intends to add to the continuing discussion on the best techniques for volatility analysis in financial time series by assessing several risk models. A comparison of the forecasting capabilities of various GARCH-type models and the Heston stochastic volatility model was also presented in the study.

LITERATURE REVIEW

Numerous econometric models have been used to forecast the volatility of crude oil prices. With an emphasis on GARCH-type models, regime-switching models, ARIMA models, and the macroeconomic ramifications of oil price volatility, this study looks at the main approaches and conclusions from earlier studies.

GARCH-Type Models for Forecasting Oil Price Volatility

Crude oil price volatility has been widely modeled and predicted using GARCH models and its extensions. Saltik *et al.* (2016) used the GARCH, IGARCH, GJR-GARCH, EGARCH, FIGARCH, and FIAPARCH models to examine the return volatility of Henry Hub natural gas and WTI crude oil over various time periods. According to their research, asymmetric and integrated GARCH models outperformed ordinary GARCH models in terms of forecast accuracy. In particular, according to Mean Square Error (MSE) and Mean Absolute Error (MAE) criteria, FIGARCH under skew Student-t performed best for one period, whereas EGARCH under the generalized error distribution was optimal for another. Similarly, Herrera *et al.* (2018) discovered that because of the strong kurtosis in oil returns, models with a Student-t distribution outperformed those with a normal distribution. They came to the conclusion that EGARCH(1,1) was better for medium-term forecasting, whereas GARCH(1,1) and RiskMetrics models performed best for short-term projections. Kutu and Ngalawa (2017) confirmed a significant negative influence of oil price shocks on the South African currency rate using the EGARCH (1,1) model. Several autoregressive models were used in other research, including Agnolucci (2009) and Ramzan *et al.* (2012), to confirm the persistence of volatility in oil return series. The necessity of proactive monetary policy interventions was highlighted by Fasanya and Adekoya (2017), who evaluated symmetric (GARCH, GARCH-M) and asymmetric (EGARCH, TGARCH) models and concluded that EGARCH was the most suitable for simulating inflation volatility.

Regime-Switching and Alternative Volatility Models

GARCH models have been investigated in a number of studies to account for structural shifts in the volatility of oil prices. Zhang *et al.* (2019) looked at both regime-switching and single-regime GARCH models and found that simpler single-regime models frequently performed better than more intricate regime-switching models. To account for long-range dependence in financial time series, Li *et al.* (2013) developed the Mixture Memory GARCH (MM-GARCH) model, which combined the conventional GARCH and FIGARCH models. According to Klein and Walther (2016), MM-GARCH models performed better in variance and value-at-risk forecasting than conventional GARCH models. An empirical investigation contrasting the MRS-GARCH (Markov Regime-Switching GARCH) model with conventional GARCH models was carried out by Zhang *et al.* (2019). Their results indicated that whereas regime-switching models improved in-sample estimates, their out-of-sample forecasting performance was not always enhanced. The significance of mean equation optimality was further highlighted by Hasanov *et al.* (2020), who showed that GARCH models with optimal mean equations generated better predictions.

ARIMA Models in Crude Oil Price Forecasting

Forecasting has made extensive use of ARIMA models in addition to GARCH-type models. When Selvi *et al.* (2018) used ARIMA to anticipate crude oil prices from 2017 to 2021, they found that prices would continue to grow, highlighting the necessity of price stability measures. In line with Selvi *et al.* (2018), Shah & Kiruthiga (2020) determined that ARIMA (0,1,4) was the best model for predicting crude oil prices. Similarly, Rodhan & Jaaz (2022) discovered that ARIMA (1,1,4) produced the most accurate forecasts after examining 375 months of WTI crude oil price data.

Macroeconomic Implications of Oil Price Volatility

One important topic of study has been how the volatility of crude oil prices affects macroeconomic factors. Using the ARCH, GARCH, and EGARCH models, Sekati *et al.* (2020) investigated how South Africa's GDP, inflation rate, and currency rates affected the price of oil globally. According to their findings, a 1% increase in each indicator had a varied impact on oil prices, with GDP and exchange rates having a positive effect and inflation having a negative one. These findings, however, were in contrast to those of Kutu and Ngalawa (2017), who discovered that shocks to the price of crude oil had a negative impact on exchange rates. Numerous studies emphasize the detrimental consequences of volatility in the price of crude oil on economic stability, especially in developing nations. According to Yildirim (2017) and Demirer *et al.* (2018), living standards are adversely affected by ongoing changes in the price of oil. Crude oil prices are more volatile than those of other non-financial assets, which adds to economic uncertainty, according to Adelman (2000) and Lipsky (2009). In their analysis of oil price volatility from an investing standpoint, Liu *et al.* (2022) concluded that no single model consistently outperformed others, supporting the use of a variety of models to increase forecasting accuracy. According to the literature evaluation, asymmetric and integrated GARCH models outperform symmetric models in terms of forecasting the volatility of crude oil prices. Regime-switching models may not always increase predicting accuracy, although they do show promise for in-sample analysis. While macroeconomic models highlight the wider economic ramifications of oil price volatility, ARIMA models are nevertheless useful for short-term price forecasting.

MATERIALS AND METHODS

Data Description and Source

The Central Bank of Nigeria website provided daily crude oil prices from January 2010 to December 2023 for modeling and forecasting purposes. Because Brent crude oil is regarded as the standard for crude oils in Europe and Africa, this information was used. Another way that crude oil is exchanged is either on its own or in relation to other forms of crude oil. Another factor in this choice was the data's accessibility during the specified time window. Continuously compounded daily stock returns are fitted to conditional variance models.

$y_t = 100(\ln k_t - \ln k_{(t-1)})$ (1)

Where k_t = current period of stock market exchange, $k_{(t-1)}$ = previous period stock market exchange, y_t = current period stock returns (stock market exchange -RT), and $\Omega_{(t-1)}$ = All stock returns up to the immediate past.

Model's Description

The models used to estimate the volatility of crude oil prices are introduced in this section. The features of the historical crude oil spot price data are used to establish the modeling approach that is used. The best models of price volatility are not universally agreed upon since energy prices have complicated characteristics. The following are the procedures used to model the volatility: examine past data to determine its characteristics; Verify if the observations are normal. Verify the series' stationarity and look for ARCH effects; The Lagrange multiplier test is used to find out whether ARCH (Autoregressive Integrated Moving-Average) effects are present, and the Augmented Dickey-Fuller (ADF) and Philips Perron (PP) tests are used to check for stationarity. Describe the estimate processes for the five GARCH type models and the Heston stochastic model that were utilized in this work to model and forecast the price and return of crude oil. The best-fitting model is then selected by comparing the results with the predicting outcomes.

GARCH-type Models

Engle (1982) created the fundamental concept of the auto regressive conditional heteroskedasticity (ARCH) model in his groundbreaking study. In the literature, the ARCH model and its later generalized versions are widely recognized for their capacity to capture the most significant stylized facts found in all volatility measures (e.g., squared log-returns, absolute log-returns, etc.), such as clustering effects, long-memory and short-memory effects, and asymmetric leverage effects. Five distinct GARCH models that were employed in this study are presented below.

Models of Volatility

The Family of Autoregressive Conditional Heteroskedasticity (ARCH) Models. Every ARCH or GARCH family model requires two distinct specifications: the mean and variance equations. According to Engel, conditional heteroskedasticity in a return series, can be modeled using ARCH model expressing the mean equation in the form:

$y_t = E_{(t-1)}(y_t) + \varepsilon_t$ (2)

Such that $\varepsilon_t = \varphi_t \sigma_t$

Equation 2 is the mean equation which also applies to other GARCH family model. $E_{(t-1)}$ is expectation conditional on information available at time t-1, ε_t is error generated from the mean equation at time t and φ_t is a sequence of independent, identically distributed (iid) random variables with zero mean and unit variance. E $\{\varepsilon_t/\Omega_{(t-1)}\}=0$; and $\sigma^2_t = \{(\varepsilon^2_t)/\Omega_{(t-1)}\}$ is a nontrivial positive valued parametric function of $\Omega_{(t-1)}$. The variance equation for an ARCH model of order q is given as:

$\sigma^2_t = \alpha_0 + \sum^q_{(i=1)} \alpha_i \varepsilon^2_{(t-1)} + \mu_t$ (3)

Where $\alpha_0 > 0$, $\alpha_i \geq 0$, i=1,…,q, and $\alpha_q > 0$

In practical application of ARCH (q) model, the decay rate is usually more rapid than what actually applies to financial time series data. To account for this, the order of the ARCH must be at maximum, a process that is strenuous and more cumbersome.

The Unconditional Kurtosis of ARCH (1)

Suppose the innovations are normal, then

$E(a_t^4 \mid F_{t-1}) = 3[\, E(a_t^2 \mid F_{t-1})\,]^2$ (4)

$= 3(\alpha_0 + \alpha_1 a_{t-1}^2)^2$,

it follows that

$Ea_t^4 = 3\alpha_0^2 (1 + \alpha_1) / [(1 - \alpha_1)(1 - 3\alpha_1^2)]$ (5)

and

$Ea_t^4 / (Eat_2)^2 = 3(1 - \alpha_1^2) / (1 - 3\alpha_1^2) > 3$ (6)

This shows that the tail distribution of at is heavier than that of a normal distribution.

Generalized ARCH (GARCH) Model

The conditional variance for GARCH (p, q) model is expressed generally as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2 \quad (7)$$

where p is the order of the GARCH terms, and q is the order of the ARCH terms, ε^2. Where $\beta_0 > 0$, $\alpha_i \geq 0$, i=1,…, q-1, j=1,…, p-1 and β_p, α_q >0. σ^2_t is the conditional variance and ε^2_t, disturbance term. The reduced form of equation 3 is the GARCH (1, 1) represented as:

$\sigma^2_t = \beta_0 + \beta_1 \varepsilon^2_{(t-1)} + \beta^2 \sigma^2_{(t-1)}$ (8)

The three parameters (β_0, β_1 and β_2) are nonnegative and $\beta_1 + \beta_2 < 1$ to achieve stationartiy.

EGARCH Model

A different that also captures the leverage is the exponential GARCH MODEL OR EGARCH

$$ln\sigma^2_{t+1} = \omega + \alpha(\varphi R_t + \gamma[|R_t| - E|R_t|] + \beta ln\sigma^2_t \quad (9)$$

Which displays the usual leverage effect if $\alpha\varphi<0$. The EGARCH model has the advantage that the logarithmic specification ensures that variance is always positive, but it has the disadvantage that the future expected variance beyond one period cannot be calculated analytically.

Weekend Effect

It is always known that days that followed a weekend or a holiday have higher variance than average day. We can try the following model:

$$\sigma_{t+1}^2=\omega+\beta\sigma_t^2+\alpha\sigma_t^2 Zt^2+\gamma IT_{t+1}, \quad (10)$$

where IT_{t+1} takes value 1 if day t+1 is a Monday, for example.

More General EGARCH

The exponential GARCH, or EGARCH model is

$$\log(\sigma_t)= \alpha_0+ \sum^q_{(i=1)} \alpha_i\, g(\epsilon_{t-1})+ \sum^p_{(i=1)} \beta_i \log(\sigma_{t-1}) \quad (11)$$

where $g(\epsilon_t)= \theta\epsilon_t + \gamma\{|\epsilon_t|-E(|\epsilon_t|)\}$

IGARCH Model

A GARCH (p, q) process is called an I-GARCH process if

$$\sum^q_{(i=1)} \alpha_i + \sum^p_{(i=1)} \beta_i=1 \quad (12)$$

The IGARCH processes are either non-stationary or have an infinite variance.

Model Selection Criteria

This section explains the model selection criteria used to select the model combination to use. To select the best fitting ARMA-GARCH models, Akaike Information Criteria (AIC) due to (Akaike, 1974), Bayesian Information Criterion (BIC) due to Schwarz Information Criterion (SIC) due to (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) due to (Hannan, 1980) and Log likelihood are the most commonly used model selection criteria. These criteria are used in this study and are computed as follows:

$$AIC (K) = -2\log L + 2k \quad (13)$$

$$BIC (K) = -2\log L + (\log N).d \quad (14)$$

$$SIC (K)= -2\log L + K \log T \quad (15)$$

$$HQC (K) = 2\log [\log T]K - 2\log L \quad (16)$$

Where K is the number of independently estimated parameters in the model. T is the number of observations; L is the maximized value of the Log-Likelihood for the estimated model defined as follows:

$$L = \prod_{t=0}^{n} \left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2} exp\left[-\sum_{t=1}^{n} \frac{(r_t-\sigma)^2}{2\sigma_t^2}\right] \quad (17)$$

$$\ln(L) = \left[\prod_{t=1}^{n} \left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2}\right] \frac{-1}{2} \sum_{t=1}^{n} \frac{(r_t-\sigma)^2}{\sigma_t^2} \quad (18)$$

Thus, given a set of estimated GARCH type models for a given set of data, the preferred model is the one with the minimum information criteria and largest log likelihood value.

A lower AIC means a model is considered to be closer to the true model. The loglikelihood is used to select the best model for estimation and forecasting. The higher the loglikelihood, the better the model. Besides this, the information criteria are also used to pick the model. A good model had the highest loglikelihood or the lowest information criteria. Therefore, a higher log likelihood translates to a low information criterion. The information criteria used in this study are the Akaike, Bayes, Shibata and Hannan-Quinn.

Forecasting performance of the five models is analysed by comparing the errors i.e., comparing the forecasted returns with realized returns. This is done by comparing the mean error (ME), mean absolute error (MAE) and root mean square error (RMSE). The lesser the errors the better the more accurate the model is in forecasting correct return for Brent Crude Oil.

$$ME = 1/n \sum^n_{(j=1)} (y_i-y^*) \quad (19)$$

$$MAE = 1/n \sum^n_{(j=1)} |(y_i-y^*)| \quad (20)$$

$$RMSE = \sqrt{(1/n \sum^n_{(j=1)} ((y_i-y^*))^2)} \quad (21)$$

RESULTS AND DISCUSSIONS

The 5112 data observations for the oil price volatility from January 2010 to December 2023 are the main emphasis of this section. To improve the accuracy of the data analysis, some processes are looked at. As can be seen from the graphing of data behavior with extensive mobility, the first result indicates that the series is not stationary.

Plot

Figure 1 shows the energy data for the daily price of crude oil in Nigeria plotted against time. The graphic shows that fluctuations in crude oil prices show clustered volatility with sporadic surges and spikes. The crude oil price plot indicates that the price of crude oil is not regularly distributed during the given period. The recession may have caused the decline in oil prices in 2016. The significant decline in oil prices earlier in 2020, particularly from March 2020 to April 2021, which was exacerbated by the Covid-19 outbreak, must also be noted. At this time, the corona virus had infected the

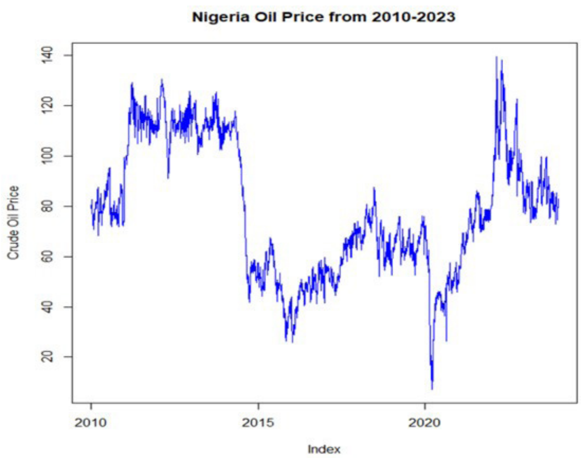

Figure 1: Plot of Crude Oil Price from 2010 to 2023

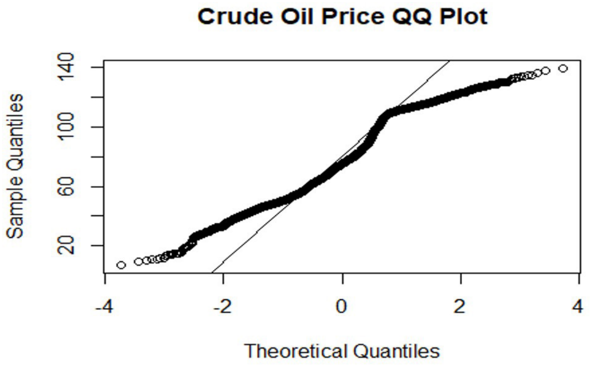

Figure 2: QQ Plot for Crude oil Sport Prices

majority of nations, and governments were beginning to enforce travel restrictions and lockdowns. The aforementioned chart demonstrates volatility clustering; rice prices rise steadily for a while before falling steadily for another. It is evident from Figure 2 that the sport prices are not distributed properly. It is clear from Figure 3 that the financial time series share characteristics. The mean reversion property is demonstrated by the fact that variance is not constant across the figures. The charts also reveal volatility clustering.

Descriptive Statistics of the Crude Oil Returns

Table 1 shows the summary statistics of the Crude Oil returns, it was shown in the result that, Crude oil return has a negative minimum return value and a standard deviation of 3.501543. From the measure of skewness, all the series for crude oil return are skewed to the left, exhibit positive excess kurtosis, which are the stylized facts observed in financial time series data. Based on the p-values of the Jarque Bera test, we reject the null hypothesis of normality for the differenced series of crude oil returns. The series have positive means and are mean-stationary since the returns are concentrated around zero as indicated in Figure 3. These series exhibit leptokurtosis as their kurtosis is greater than the normal kurtosis value of 3. Time plots are used to determine the observable characteristics of the returns as presented in Figure 3.

Table 1: Summary Statistics

Statistics	**Oil Returns**
Mean	0.0000009
Standard Error	0.475410198
Median	0.07831885
Standard Deviation	3.501543
Sample Variance	0.002704771
Kurtosis	70.87588
Skewness	-1.537876
Jarque-Bera statistics	1072872
Minimum	-0.6604506
Maximum	0.588928

Source: Author's estimation using R 4.4.1

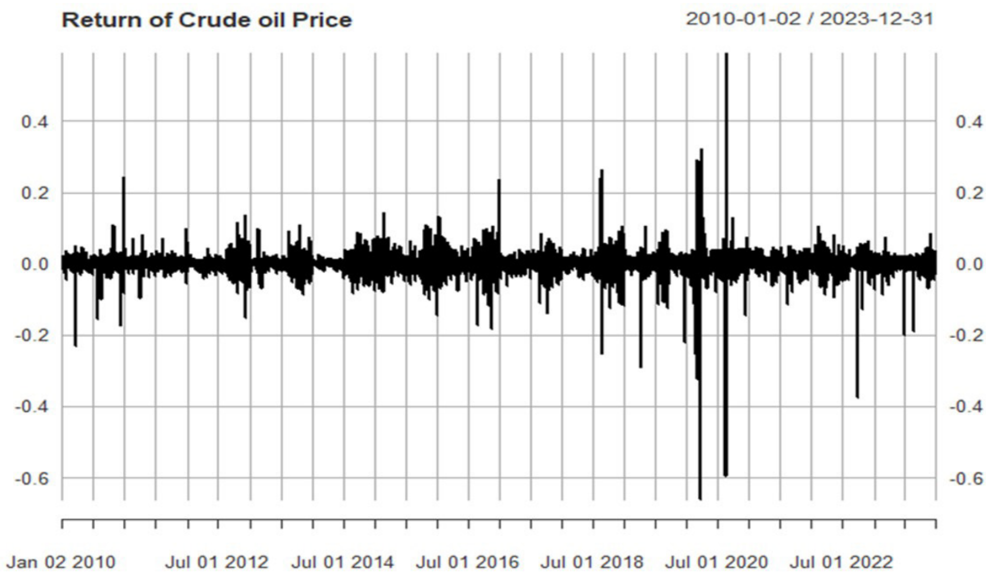

Figure 3: Plot Return of Crude Oil

Testing for Normality

To determine whether the return series is normally distributed, the normal QQ-plot is utilized to examine the distributional features. A scatter plot of a specific distribution is represented by the typical QQ-plot. The evidence against the null hypothesis that the distribution is normal increases with the degree of deviation from this line. With a few outliers (those that appear farther from the normal line) that can be interpreted as the heavier tails in the preceding image, Figure 4 illustrates that returns are generally normally distributed. This plot demonstrates that the returns in this study can be modeled using a normal distribution, but the heavy tails would not be addressed. In order to address the heavy tail, the student t distribution which is known to be capable of capturing heavy tails is now taken into consideration. The majority of the previously noted outliers have been eliminated by the plot in Figure 5. Because the T-distribution can catch heavier tails, as shown by our research above, it was added to Table 2. At the 5% level of significance, the high Jarque-Bera statistics value provides evidence that the null hypothesis of normalcy can be rejected. This conclusion is further supported by the large excess kurtosis and negative skewness. Similar to the normal distribution, the student t distribution is bell-shaped and symmetrical. Nonetheless, the t-distribution is more likely to yield values that deviate significantly from its mean due to its thicker tails. We decided to use the normal distribution and t-distribution to suit the volatility models because we are unable to completely dismiss the normal distribution in this investigation.

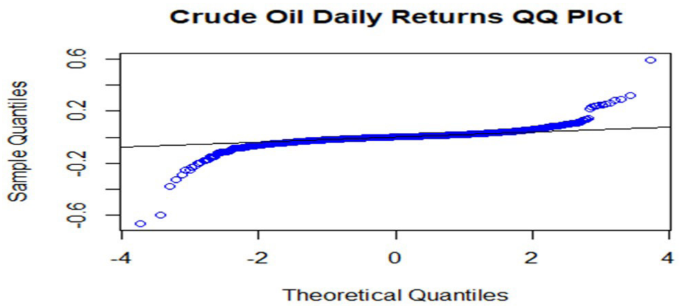

Figure 4: QQ Plot for Normal Distribution

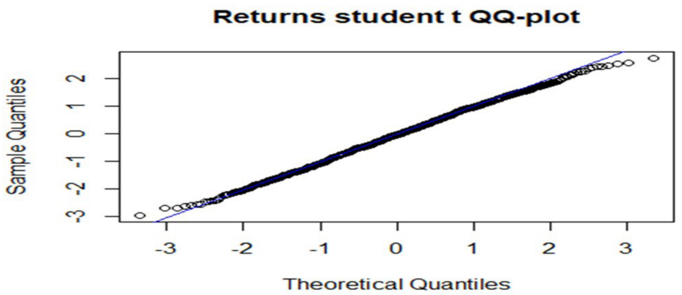

Figure 5: QQ Plot for Student t Distribution

Table 2: Jarque Bera Test, Skewness, Kurtosis for Checking Normality

	X-squared	p-value
Jarque Bera Test	1072872	0.0000
Skewness	-1.537876	
Kurtosis	70.87588	

Source: Author's estimation using R *4.4.1*

Volatility Clustering

Return series fail to follow the financial stylized facts. Hence improving ARIMA model by using GARCH (Generalized Auto Regressive Heteroskedasticity) types model.

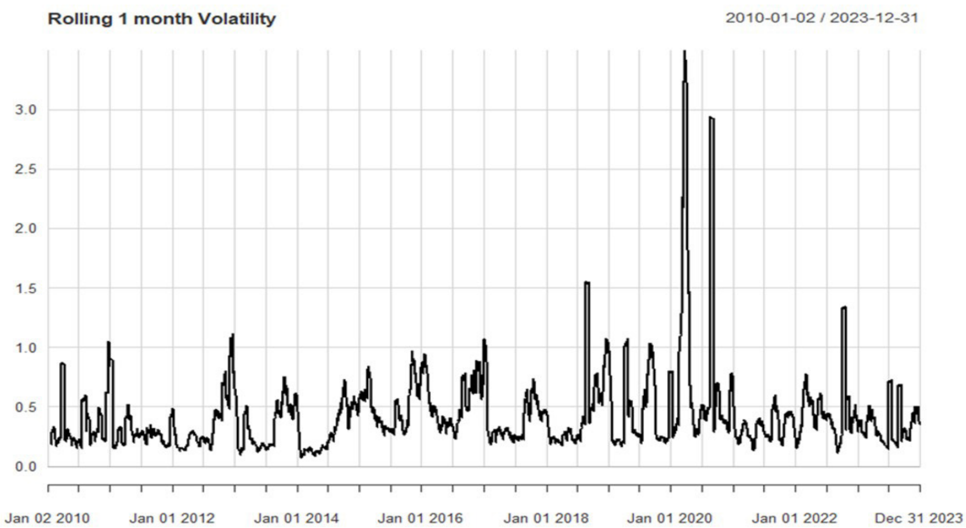

Figure 6: Volatility Clustering Plot

Stationary Tests

To investigate whether the return series are stationary, the Augmented Dickey-Fuller (ADF) test was applied. The hypothesis was such that: H0: Non stationary versus H1: Stationary. The p-value is < 0.05. This allows the rejection of the null hypothesis. To confirm the results above, we use the Philips Perron (PP) test which is a non-parametric correction to ADF to account for autocorrelation associated with breaks in the data. As the test values are lower than the critical values by choosing the 1% confidence level, it can be certainly confirmed that the null hypothesis is rejected and the Crude Oil returns series is stationary.

Table 3: Stationary Test

Augmented Dickey Fuller (ADF)	**Estimate**	**t-Statistic**	**Prob.**	**Test critical values (1% level)**
Intercept	4.26797	2.693	0.00768	-3.46
z.lag.1	-0.05367	-2.797	0.00567	
z.diff.lag	0.30584	4.509	1.11e-05	
F-Statistics	12.54			
Phillips-Perron Unit Root Test				
Dickey-Fuller = -70.883	P_ value = 0.01			

Source: Author's estimation using R *4.4.1*

Presence of Volatility

We check the presence of volatility using time series plot for log returns, square returns, and absolute returns. This part of investigating the Crude Oil return series represents a special place in quantitative research, because useful information could be found for testing the stationary hypothesis, heteroskedasticity effect and by the descriptive statistical information about the average, variance, asymmetry indicators and the type of distributions, before applying the desired models of estimating and predicting the conditional volatility of these returns. From this point of view, this study aligned with the preferred tools of analysis the oil return series used by Yildirim (2017); Zhang *et al.* (2019); Haque and Shaik (2021) or Mohammadi and Su (2010).

The next step was to identify the presence of ARCH terms, thus testing the level of increased probability of ARCH effects (q). This plays an important role in the use of ARCH-GARCH models and the manner in which the analysed time series can be estimated and fitted by these models (Yi *et al.*, 2021; Sekati *et al.*, 2020; Oyuna and Yasbin, 2021).

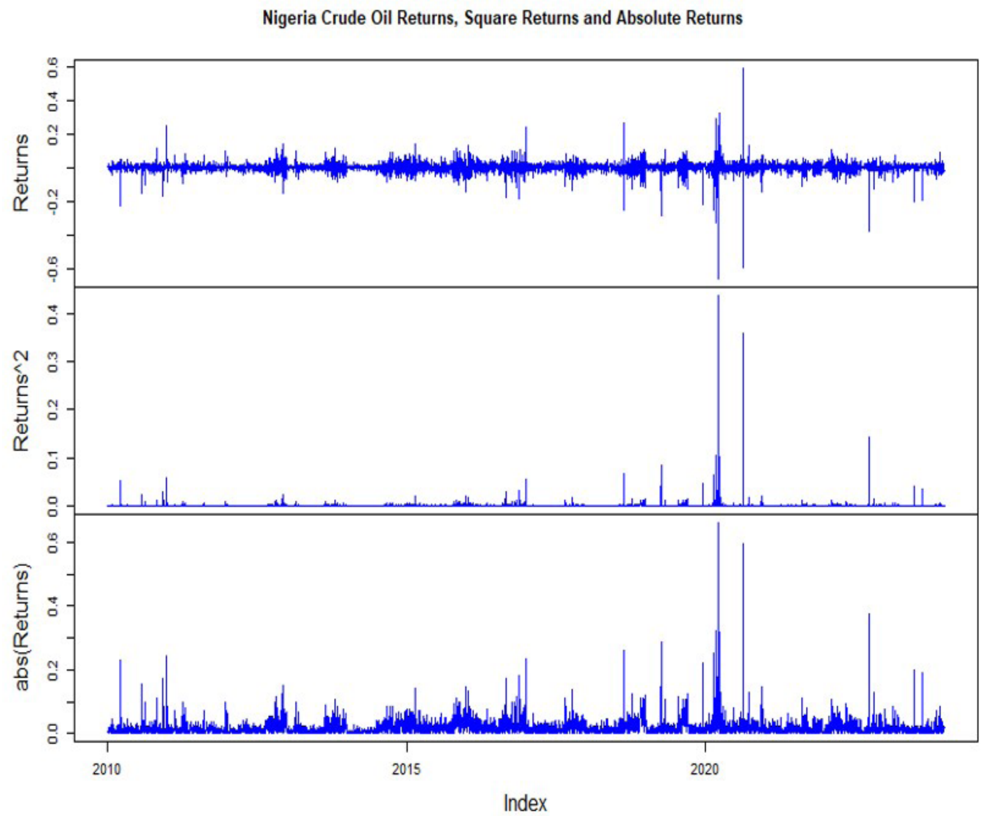

Figure 7: Plot of Crude oil Price Volatility

Presence of ARCH (q) Effects

Figure 7 sows the returns (square and absolute) show high level of auto-correlation. We double checked present of auto-correlation in square returns by applying Ljung-Box test on square returns and the p-value (0.0000) is < 0.05 which indicated that the data is not independent. The null hypothesis was rejected which means auto correlation is present. The Lagrange multiplier (LM) test by ENGLE (1982) was applied to the residuals of simple time series models. The ARCH-LM tests results provided strong evidence for rejecting the null hypothesis as shown in Table 4. Implicitly, the associated p-value, which is lower than the 5% confidence level indicates that ARCH effects exist; hence, an ARCH or GARCH model should be employed in modeling the return time series. This fact is also observed at the p-value of lagged squared error terms (0.000) that is less than 5% confidence level and indicates the same type of conclusion: the presence of ARCH effects in the oil return series.

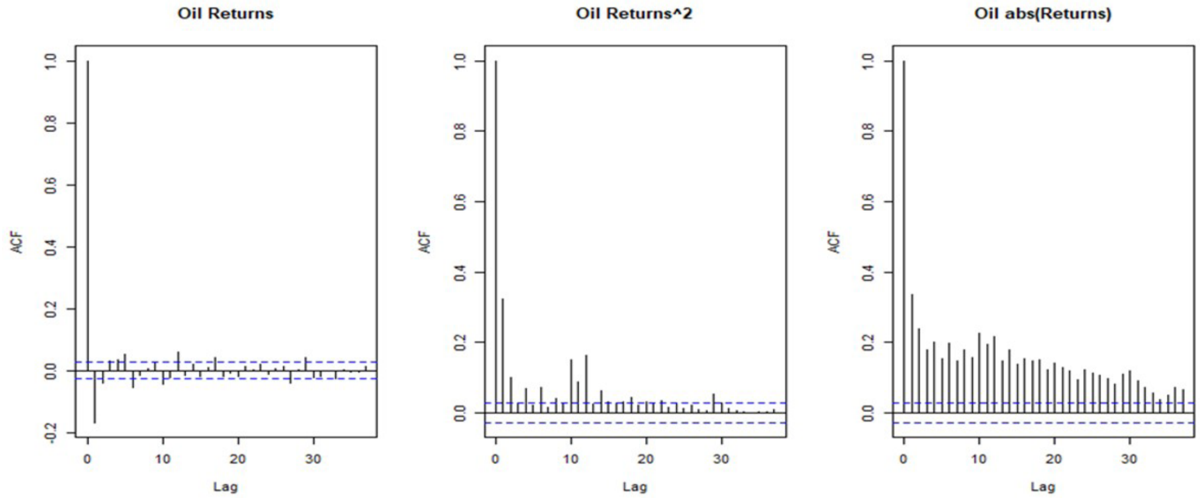

Figure 8: Checking Autocorrelation in Return

Table 4: ARCH Effect Test

Variable	Coefficient	Std.Error	Prob.
C	0.90826	0.12440	0.0000
$\mu^2_{(t-1)}$	0.35307	0.04198	0.0000
Chi-squared	761.94	2.2e-16	

Source: Author's estimation using R *4.4.1*

GARCH Model

To get a precise assessment of the volatility of crude oil, this part used symmetrical and various asymmetrical GARCH models of the normal and student's t distribution innovation. The GARCH model (p, q) was used to examine two equations: the conditional variance and the conditional mean. With the exception of mu, which is not significant, Table 5 visual representation suggests that all of the coefficients are positive and highly statistically significant (5% p-value). At 11981.45, the log probability return is positive. The respective values for the AIC, BIC, SC, and HQ are -4.6856, -4.6792, -4.6856, and -4.6834. When compared to other models, these values are all fairly near to one another and serve as criteria for determining which model is best. Additionally, our findings demonstrated that volatility shocks endure, as indicated by the sum of the ARCH and GARCH coefficients, which is a common finding in recent research (Brandt and Gao, 2019; Miao *et al.*, 2017; Yildirim, 2017). With regard to Table 6, TGARCH (1,1) model results, every parameter aside from mu, is positive and statistically significant at the 5% p-value. This indicates that the TGARCH (1,1) model's requirements are met. Table 7 details the outcomes received for this model. When calculating the average return on crude oil, the TGARCH (1,1) model is novel since it accounts for volatility (either as conditional variance or conditional standard deviation). Thus, like earlier research (Yildirim, 2017; Kutu and Ngalawa, 2017; Neshat *et al.*, 2018), the TGARCH (1,1) model can show how investors' risk aversion affects the world's oil price fluctuations. The individual outcomes of the IGARCH (1,1) model are shown in Table 8. Because of its popularity and capacity to analyze the asymmetric reaction to different market shocks, it is used to quantify the risk associated with any kind of financial or non-financial asset. With the exception of mu, which is not, all of the coefficients are positive and statistically significant at the 1% level based on the parameters. The stability of the IGARCH (1,1) model is further confirmed by the statistical significance of the computed parameters at the level of conditional variation (Saltik *et al.*, 2016; Mohammadi and Su, 2010; Herrera *et al.*, 2018). Additionally, 11981.46 is a positive value for the IGARCH loglikehood. The AIC, which has a value of -4.6860, was computed using the IGARCH (1,1) model.

GARCH (1,1)

Table 5: The main results about the estimated conditional variance by using the GARCH (1,1) models

Variables/Parameters	Coefficients	Std. error	Prob.
Mu	0.000089	0.000247	0.71926
Omega	0.000032	0.000006	0.00000
Alpha	0.211262	0.026751	0.00000
beta1	0.787717	0.021613	0.00000
Shape	3.638566	0.189317	0.00000
Log likelihood	11981.45		
Akaike information criterion (AIC)	-4.6856		
Bayes information criterion (AIC	-4.6792		
Shibata criterion (SC)	-4.6856		
Hannan-Quinn criterion (HQ)	-4.6834		

Source: Author's estimation using R *4.4.1*

EGARCH (1,1)

Table 6: The results for EGARCH (1,1) model

Variables/Parameters	Coefficients	Std. error	Prob.
Mu	0.000089	0.000082	0.27958
Omega	-0.480951	0.063635	0.00000
alpha 1	0.075537	0.011234	0.00000
beta1	0.925548	0.009301	0.00000
gamma1	0.303257	0.023440	0.00000
Log likelihood	10963.96		
Akaike information criterion (AIC)	-4.2875		
Bayes information criterion (AIC	-4.2811		
Shibata criterion (SC)	-4.2875		
Hannan-Quinn criterion (HQ)	4.2853		

Source: Author's estimation using R *4.4.1*

The Results from TGARCH (1,1) Model

Table 7: TGARCH (1,1) Model

Variables/Parameters	Coefficients	Std. error	Prob.
Mu	0.000089	0.000249	0.720996
Omega	0.000032	0.000006	0.000000
Alpha1	0.164108	0.028883	0.000000
beta1	0.786630	0.021824	0.000000
gamma1	0.096304	0.036656	0.008608
Shape	3.633869	0.190538	0.000000
Log likelihood	11985.2		
Akaike information criterion (AIC)	-4.6867		
Bayes information criterion (AIC	-4.6790		
Shibata criterion (SC)	-4.6867		
Hannan-Quinn criterion (HQ)	-4.6840		

Source: Author's estimation using R *4.4.1*

IGARCH (1,1) Model

Table 8: The results from IGARCH (1,1) model

Variables/Parameters	Coefficients	Std. error	Prob.
Mu	0.000089	0.000247	0.71896
Omega	0.000032	5.45374	0.00000
Alpha1	0.212413	0.021528	0.00000
beta1	0.787587	NA	NA
	3.629988	0.147572	0.00000
Log likelihood	11981.46		
Akaike information criterion (AIC)	-4.6860		
Bayes information criterion (AIC	-4.6809		
Shibata criterion (SC)	-4.6860		
Hannan-Quinn criterion (HQ)	-4.6842		

Source: Author's estimation using R *4.4.1*

FIGARCH (1,1) Model

Table 9: The results from FIGARCH (1,1) model

Variables/Parameters	Coefficients	Std. error	Prob.
mu	0.000089	0.000247	0.719240
omega	0.000034	0.000008	0.000036
Alpha1	0.135460	0.055987	0.015543
beta1	0.745468	0.053460	0.000000
delta	0.876198	0.067964	0.000000
shape	3.652282	0.149356	0.000000
Log likelihood	11985.19		
Akaike information criterion (AIC)	-4.6867		
Bayes information criterion (AIC	-4.6790		
Shibata criterion (SC)	-4.6867		
Hannan-Quinn criterion (HQ)	-4.6840		

Source: Author's estimation using R *4.4.1*

The Investigation Analysis of the Applied GARCH Type Models

The literature (Yi *et al.*, 2021; Yildirim, 2017; Er and Fidan, 2013; Kulikova and Taylor, 2013; Zhang and Wang, 2015; Aye *et al.*, 2014) outlines a number of requirements that each model used must meet, beginning with the specific hypotheses in applying the ARCH-GARCH approaches. Accordingly, the models should have the fewest parameters, the highest Log Likelihood ratio, the lowest Schwarz Information Criteria, significant ARCH and GARCH parameters, and neither autocorrelation nor heteroskedasticity in the residual or errors terms. Starting from this assumption, we concentrated on the analysis and diagnosis of the five models that were used: GARCH (1,1), IGARCH (1,1), EGARCH (1,1), TGARCH, and FIGARCH (1,1). The specific goal was to determine which model was best suited for estimating the conditional variance, or which model had the most criteria completed. The results of the ARCH-LM and Durbin-Waston tests, which do not account for heteroskedasticity or autocorrelation, are displayed in Table 10.

Table 10: The results of evaluation models (Test for Residuals)

Variables/Parameters	Std. error	Prob.
MODEL	**ARCH-LM TEST**	**DURBIN-WASTON TEST**
GARCH (1,1)	0.3758 (0.5398)	1.999914
EGARCH (1,1)	0.2767 (0.5988)	1.999906
TGARCH (1,1)	0.4385 (0.5078)	1.999936
IGARCH (1,1)	0.1128 (0.7370)	1.999911
FIGARCH (1,1)	0.002673 (0.9588)	1.999983

Source: Author's estimation using R *4.4.1*

It can be stated categorically state that the models (GARCH, IGARCH, TGARCH, and FIGARCH) that were employed have been successful in meeting the error series' residual requirements. The hypothesis of homoskedasticity is thus validated since the probability related to the ARCH-LM test are higher than the 5% (0.05) threshold that the existence of ARCH (q) effects in the residual series is denied and the errors are uniformly distributed. The Durbin-Waston Test results, which were about equivalent to 2.00, verified that there was neither autocorrelation nor serial correlation on the residual series. In addition, Table 10 shows that the EGARCH (1,1) model is the best accurate model for estimating the conditional variance of the Crude Oil return series. This model is completely compliant and genuinely respects the primary restrictive constraints imposed by the literature in contrast to the GARCH (1,1), TGARCH, IGARCH (1,1), and FIGARCH (1,1) models. Studies by Yang *et al.* (2020), Saltik *et al.* (2016), and Yildirim (2017) that use asymmetrical and non-parametric GARCH models to evaluate crude oil volatility across time are positioned in a similar manner.

The Best Estimated Model for Selection

The best model is chosen based on its logarithm maximum likelihood function value. The better the model, the lower the values of the information criteria. According to Tables 11 and 12 above, the EGARCH model has the lowest information criteria values for the t-distribution and the highest log likelihood value for the normal distribution. But, because it has a greater log probability value and fewer information criteria than the normal distribution, the t-distribution matches the data better. This demonstrates that, out of all the models examined, the EGARCH-t distribution fits the data the best.

Table 11: Loglikelihood and Information Criteria values normal dist

MODEL	ARCH significant?	GARCH significant?	LOG LIKELIHOO	Akaike IC
GARCH	**Yes**	**Yes**	**10943.67**	**-4.2800**
EGARCH	Yes	Yes	10966.96	-4.2889
TGARCH	Yes	Yes	10966.84	-4.2887
IGARCH	Yes	Yes	10939.28	4.2787
FIGARCH	Yes	Yes	10842.81	-4.2401

Source: Author's estimation using R 4.4.1

Table 12: Loglikelihood and Information Criteria values—student's dist

MODEL	ARCH significant?	GARCH significant?	LOG LIKELIHOOD	Akaike IC
GARCH	**Yes**	**Yes**	**11981.45**	**-4.6856**
EGARCH	Yes	Yes	12022.3	-4.7012
TGARCH	Yes	Yes	11985.2	-4.6867
IGARCH	Yes	Yes	11981.46	-4.6860
FIGARCH	Yes	Yes	11985.19	-4.6867

Source: Author's estimation using R 4.4.1

Forecast

The forecast of future conditional volatility of the crude oil return series was calculated following the diagnostic analysis of GARCH models. Demirer *et al.* (2018), Escribano and Valdes (2017), and Ahmed and Shabri (2014) used the data from each volatility model with the lowest values of Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE), and Root Mean Square Error (RMSE) to determine the optimal forecasting model. In a more unpredictable, challenging, and stressful time brought on by the COVID-19 Pandemic, there was a greater need to monitor the persistence and fluctuating and oscillating movement of crude oil volatility, which is why the 2010–2023 timeframe was chosen for analysis. The overall assessment of five GARCH-type models using the RMS and MAE error metrics is shown in Table 13. According to the findings, the EGARCH model outperforms the GARCH, IGARCH, FIGARCH, and TGARCH models in terms of forecast accuracy. The

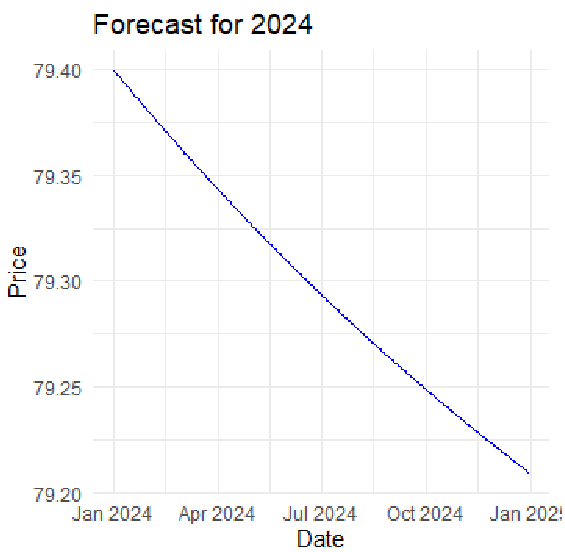

Figure 9: Forecast of Crude Oil Prices Using EGARCH

outcome demonstrated that each of the four models yields outcomes that are comparatively satisfactory. This suggests that none of the models' significance in predicting crude oil returns can be disputed. But the EGARCH model performs better than all the others, with the IGARCH and GARCH models coming in second and third, respectively. This demonstrates that the basic GARCH model is still useful for predicting crude oil prices, despite the creation of additional GARCH extension models. This outcome makes it abundantly evident that the model that performs best when evaluating forecast performance is also the most appropriate for modeling crude oil returns in Nigeria. This is consistent with (Dana, 2016) results that the best model for forecasting is also the one that fits the data the best.

Table 13: Forecast Results (2010-2023)

MODEL	RMSE	MAE	Rank
GARCH (1,1)	0.053350424	0.0256712	3
EGARCH (1,)	0.05335017	0.0254007	1
TGARCH (1,1)	0.05350445	0.02542721	4
IGARCH (1,1)	0.05335032	0.02540104	2
FIGARCH (1,1)	0.05337822	0.02562058	5

Source: Author's estimation using R *4.4.1*

Approximation of Volatilities and the Model Parameters

By applying the Parkinson extreme value method, we approximate daily volatilities. The Parkinson extreme value method is a best estimator of volatility than the traditional volatility measure (Parkinson, 1980). Hence, by using this method, we find the daily and annual crude oil price volatilities (σ) with respective variances (σ2) for each year.

Table 14: Approximation of the Crude Oil Volatilities, 2010–2023

Time interval	Number of Trading days	Minimum Price	Maximum price	Annual Volatility	Annual Variance	Daily Volatility	Daily Variance
4.1.2010-31.12.2010	252	64.78	91.47	0.1280	0.016	0.00006	4.228E−09
3.1.2011-29.12.2011	252	75.40	113.40	0.0226	0.001	0.00009	8.076E−09
3.1.2012-31.12.2012	252	75.40	109.39	0.1316	0.017	0.00069	4.717E−09
2.1.2013-27.12.2013	252	86.55	110.61	0.0792	0.006	0.00025	6.184E−10
3.1.2014-31.12.2014	249	53.45	107.96	0.1145	0.132	0.00053	2.773E−09
2.1.2015-30.12.2015	252	34.55	61.35	0.2030	0.041	0.00016	2.674E−08
4.1.2016-30.12.2016	252	26.19	54.01	0.2111	0.045	0.00018	3.126E−08
3.1.2017-29.12.2017	250	42.48	60.45	0.1071	0.012	0.00005	2.106E−09
2.1.2018-31.12.2018	249	44.48	77.41	0.1367	0.019	0.00008	5.638E−09
4.1.2019-09.12.2019	250	46.31	66.23	0.1483	0.022	0.00009	8.757E−09
2.1.2020–28.12.2020	252	50.57	86.07	0.1282	0.0164	0.00006	4.25212E–09
2.1.2021–30.12.2021	255	35.26	66.33	0.1768	0.0312	0.00012	1.50102E–08
4.1.2022–30.12.2022	255	27.10	54.97	0.2004	0.04012	0.00016	2.48212E–08
3.1.2023–30.12.2023	256	88.69	128.14	0.0995	0.0099	0.00004	1.58121E–09

Source: Author's estimation using R *4.4.1*

Simulations with Heston stochastic volatility model was performed for crude oil data by focusing on the logarithmic crude oil price behavior. The Heston model addresses well all kinds of fat-tails properties in the daily price return distributions under various market circumstances. Initially, the parameters were computed from the observation from January 2010 to December 2023. To forecast, we used the data from 02.01.2023 to 30.10.2023. That is, for t = 1 to N is used to estimate the parameters and then forecast N +1. The Euler–Maruyama numerical method is employed in this study to simulate the Heston model. The computed parameters are presented in Table 15.

Table 15: Computed parameters for Euler–Maruyama

Parameter	Symbol	Value
Initial price	Y_0	46.31
Initial volatility	V_0	2.3×10^{-4}
Vol-volatility	σ	9.0×10^{-5}
long-run variance	θ	8.8×10^{-9}
Reversion rate	β	2.95×10^{-3}
Mean log-return	μ	4.94×10^{-4}

Source: Author's estimation using R *4.4.1*

Table 16: The Error Analysis, The Euler-Maruyama Scheme

Step size (Δt)	Error
0.0010	0.000564
0.1010	0.010153
0.2010	0.011524
0.3010	0.010771
0.4010	0.012742
0.5010	0.013142
0.6010	0.012421
0.7010	0.012101
0.8010	0.010621
0.9010	0.011283

Source: Author's estimation using R *4.4.1*

Table 16 presents the error analysis for the Heston model by using the Euler–Maruyama method. Results obtained are compared with results of GARCH-type models presented in Table 11. The model with smallest error compared implies that it is more accurate to estimate the model. Comparing results of Tables 13 and 16, results in general show that the Heston model approximation presents small errors if compared with the improved GARCH models. Thus, it can be concluded that the Heston model forecast better crude oil price volatility than the counterpart models.

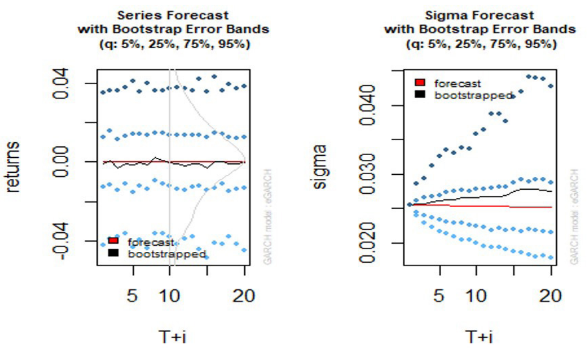

Figure 10: Forecasting using Bootstrap

Discussion

The performance of the GARCH, EGARCH, IGARCH, TGARCH, FIGARCH, and Heston models in simulating and predicting the volatility of crude oil returns using data from Nigeria is compared in this paper. A deviation from normality is shown by the return series' positive excess kurtosis and negative skewness. The null hypothesis of normality is rejected for every differenced series of crude oil return, according to the Jarque-Bera test results. Additionally, as Figure 2 shows, the series exhibits mean stationarity, with returns centered around zero. The adoption of the Student's t-distribution to account for heavy tails is further supported by the presence of leptokurtosis, when kurtosis values are higher than the typical threshold of 3. Additionally, the ARCH Lagrange Multiplier test and visual inspections are used to confirm the evidence of mean reversion and heteroscedasticity. A variety of GARCH-type models were estimated in order to choose the best model for volatility. The Akaike Information Criterion (AIC), forecasting accuracy metrics, and log-likelihood values were adopted in choosing the best model. Better model fit is indicated by a higher log-likelihood value, but more model efficiency is suggested by a lower AIC. The EGARCH model performed the best among the competing models, obtaining the lowest AIC (-4.7012) and the greatest log-likelihood value (12022.3). Additionally, the EGARCH model demonstrated greater prediction accuracy with the lowest Mean Error (0.0254007) and Root Mean Squared Error (RMSE) (0.05335017). Thus, among the GARCH-type models analyzed, the EGARCH model with Student's t-distributed innovations is found to be the best model for predicting the volatility of Nigeria's crude oil return. The study assesses the Heston model, which successfully represents the fat-tailed characteristics of daily return distributions under various market scenarios, in addition to the GARCH family models. Data from January 2010 to December 2023 were used to estimate the model parameters, and data from January 2023 to December 2023 were used to provide forecasts. The model was simulated using the Euler-Maruyama numerical method, and the calculated parameters are shown in Table 13. The Heston model provided results with less forecasting errors than the EGARCH model, according to a comparison of forecasting accuracy (Tables 12 and 14). This result implies that the Heston model outperforms the EGARCH model in forecasting crude oil return volatility, even though the latter offers robust volatility modeling. As a result, the Heston model outperforms the GARCH-type models taken into consideration in this study and is the recommended option for predicting the volatility of crude oil prices.

CONCLUSION

In this study, the Heston stochastic model and GARCH type models were used to estimate the volatility of crude oil data. Based on a number of factors, including the lowest AIC value of -4.7012 and the greatest log-likelihood value of 12022.3, it was discovered that the student's t test of the EGARCH (1,1) model is the best fitted model to estimate volatility of crude oil data among other GARCH type models. Even though EGARCH was the best-fitting model in our study, this does not necessarily mean that it is the best model for estimate and volatility forecasting in other contexts. Based on the lowest MAE value of 0.0254007 and RMSE of 0.05335017 for Nigeria Crude Oil data, the EGARCH (1,1) model likewise seems to be the most effective GARCH model for predicting. This demonstrates how crucial it is to evaluate the model's performance at every level, including predicting performance and best fitted model, in order to select the optimal model. The Heston volatility model was also approximated using the Euler–Maruyama approach. Additionally, the approximation results of the modified GARCH models were compared with the outcomes of the Heston model. The findings showed that the Heston stochastic volatility model outperforms the selected GARCH models in predicting the volatility of crude oil. As a result, it is concluded that, the Heston model in this study is the most effective for volatility forecasting and estimate.

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